Math 341: Quiz 4

Name:

True/False: If the statement is true, give a brief explanation of why it is. If the statement is false, give a counterexample.

1. [3] The function $T : \mathbb{R}^2 \to \mathbb{R}^2$ define by $T([x\ y]^T) = [\cos x\ \ \sin y]^T$ is linear.

2. [3] In an abstract vector space, $c\vec{0} = \vec{0}$ for any scalar c.

3. [3] Let $\vec{v_1}, \vec{v_2} \in \mathcal{P}_2$ where $\mathcal{P}_2$ is the vector space of all polynomials with degree less than or equal to 2. If $\vec{v_1} \neq \vec{v_2}$, then $\text{span}\{\vec{v_1}\} \neq \text{span}\{\vec{v_2}\}$.

4. [3] If $U = \text{span}\{\vec{v_1}, \vec{v_2}\}$ and $W = \text{span}\{\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}\}$, where $\vec{v_1}, \vec{v_2}, \vec{v_3}$, and $\vec{v_4}$ are all from some vector space V, then W is strictly larger then U.

Free Response: Show all your work and justify your steps. No credit is given for the correct answer with no justification.

5. Let $V = \{\vec{T} : \mathbb{R}^3 \to \mathbb{R}^2 | \vec{T}$ is a linear function$\}$. The elements in our vector field are thus *linear maps*. Define vector addition and scalar multiplication by the following:

$$(\vec{T}+\vec{R})\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \vec{T}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) + \vec{R}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) \qquad (\alpha\vec{T})\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \alpha\left(\vec{T}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)\right)$$

where $\alpha \in \mathbb{R}$, and $\vec{T}, \vec{R} \in V$.

It is given that V forms a vector space under the defined operations above. In particular note that the zero vector is the linear map defined by $\vec{0}([x\ y\ z]^T) = [0\ 0]^T$ for all $[x\ y\ z]^T \in \mathbb{R}^3$.

(a) [2] Given a subset W of V, what properties must you exhibit to verify that W is a subspace of V?

(b) [6] Show that

$$S = \{\vec{T} \in V | \exists\, A \in \mathrm{Mat}_{2x2}(\mathbb{R}) \text{ so that } \vec{T}\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = A\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) \text{ for all } \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) \in \mathbb{R}^3\}$$

forms a subspace of V.