# Math 341: Quiz 3

Name: Key

True/False: If the statement is true, give a brief explanation of why it is. If the statement is false, give a counterexample. Let $A$ and $B$ be matrices, and $c$ be a scalar.

1. [3] The function $f : \mathbb{R} \to \mathbb{R}$ defined by the rule $f(x) = x + 1$ is a linear function. Hint: write down what it means for a function to be linear.

note. a linear function is different than a line.

Recall a function $f$, is linear if it maintains linear combinations ie $f(a\vec{x}+b\vec{y}) = af(\vec{x}) + bf(\vec{y})$ for scalars $a, b$ & vectors $\vec{x}$ & $\vec{y}$. Here our vectors are only $1\times 1$ matrices.

$$3 = 2+1 = f(2) = f(1+1) \neq f(1)+f(1) = (1+1)+(1+1) = 2+2 = 4$$

False.

not graded this time

2. [3] A matrix is symmetric if and only if it is Hermitian. (Recall that a Hermitian matrix is a matrix that is equal to its conjugate transpose.)

Both directions are false

answer looking for ex get one.

Sym $\not\Rightarrow$ Hermitian

$A := \begin{bmatrix} 1 & -i \\ -i & 2 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & -i \\ -i & 2 \end{bmatrix}$ $A^* = \begin{bmatrix} 1 & i \\ i & 2 \end{bmatrix}$

Hermitian $\not\Rightarrow$ Sym

$B = \begin{bmatrix} 1 & -i \\ i & 2 \end{bmatrix}$

$B^T = \begin{bmatrix} 1 & i \\ -i & 2 \end{bmatrix}$ $B^* = \begin{bmatrix} 1 & -i \\ i & 2 \end{bmatrix}$

3. [3] An upper triangular and symmetric matrix is a diagonal matrix.

True: Let $A$ be an upper triangular & symmetric matrix. Since it is symmetric $a_{ij} = a_{ji}$ for all appropriate $i, j$. Since $A$ is upper triangular $a_{ij} = 0$ for $i > j$ thus $a_{ij} = a_{ji} = 0$ for $i > j$. Thus $A$ is diagonal.

4. [3] If $A$ and $B$ are invertible matrices, then $A + B$ is invertible.

False

let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Both $A$ & $B$ are invertible but $A+B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ which is not.

Free Response: Show all your work and justify your steps. No credit is given for the correct answer with no justification.

5. [4] Let $A = \begin{bmatrix} 2 & -2 & 1 \\ 0 & 2 & 0 \\ 2 & 0 & 1 \end{bmatrix}$. Find $A^{-1}$ if it exists. Remember, you can verify your answer is correct.

$$\left[\begin{array}{ccc|ccc} 2 & -2 & 1 & 1 & 0 & 0\\ 0 & 2 & 0 & 0 & 1 & 0\\ 2 & 0 & 1 & 0 & 0 & 1\end{array}\right] \xrightarrow{R_3-R_1=NR_3} \left[\begin{array}{ccc|ccc} 2 & -2 & 1 & 1 & 0 & 0\\ 0 & 2 & 0 & 0 & 1 & 0\\ 0 & 2 & 0 & -1 & 0 & 1\end{array}\right] \xrightarrow{R_3-R_2=NR_3}$$

$$\left[\begin{array}{ccc|ccc} 2 & -2 & 1 & 1 & 0 & 0\\ 0 & 2 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & -1 & -1 & 1\end{array}\right]$$

The left side is not equivalent to $I_3$
$\Rightarrow A$ is not invertible

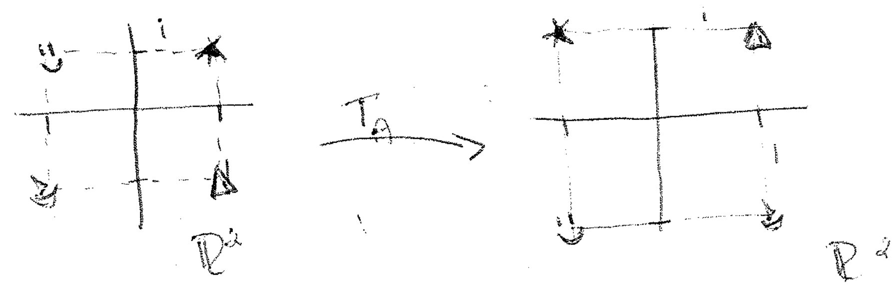
2 calculations → 1pt.

- another way -

$$\det(A) = \begin{vmatrix} 2 & -2 & 1 \\ 0 & 2 & 0 \\ 2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 2 & -2 & 1 \\ 0 & 2 & 0 \\ 0 & 2 & 0 \end{vmatrix} = \begin{vmatrix} 2 & -2 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

6. [4] Determine the effect of the matrix operator $T_A$ on the points $(\pm1, \pm1)$ where $A$ is $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Plot the ~~images~~ of the squares with the corners $(\pm1, \pm1)$, and explain in words what $T_A$ is doing to the points in $\mathbb{R}^2$.

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$




exp.
true by 3
not more than $(\pm1,\pm1)$
false 2
calc 2

$T_A$ rotates the points $\pi/2$ radians counterclockwise about the origin.

Note $A = F(i)$ where $F: \mathbb{C} \to Mat_{2\times2}(\mathbb{R})$, $a+ib \longmapsto \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$. So $T_A = T_{F(i)}$

$\Rightarrow$ this is how $i$ would act on $\mathbb{R}^2$.



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