Quiz 2
Math 341

Name: KEY

True/False: If the statement is true, give a brief explanation of why it is. If the statement is false, give a counterexample. Let A and B be matrices, and c be a scalar.

1. [3] If a linear system is inconsistent, then the rank of the augmented matrix exceeds the number of unknowns.

False

$$A := \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & * \\ 0 & 1 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

note $rk(A) = 3$ but the # of unknowns is 4.

2. [3] If $cA = 0$, then either $c = 0$ or $A = 0$.

True. Assume $c \neq 0$. We can scale both sides by $1/c$

$\Rightarrow \frac{1}{c}(cA) = \frac{1}{c}0$.

Associativity $\Rightarrow \frac{1}{c}(cA) = (\frac{1}{c} \cdot c) \cdot A = A$.

Scaling $\frac{1}{c} 0 = 0$.

So we have $A = 0$.

3. [3] If $AB = 0$ then $A = 0$ or $B = 0$.

False

$$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 1-1 & 1-1 \\ -1+1 & -1+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

4. [3] If both AB and BA are defined then $AB = BA$.

False let $A = [0, 0]$ & $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

Then $\underset{(1\times 2)}{A} \cdot \underset{(2\times 1)}{B} = [0,0]\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \underset{(1\times 1)}{[0]}$

but $\underset{(2\times 1)}{B} \cdot \underset{(1\times 2)}{A} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}[0,0] = \underset{2\times 2}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}$

Two matrices that are different sizes cannot be equal

Free Response: Show all your work and justify your steps. No credit is given for the correct answer with no justification.

Next time I'll make a more involved lin sys

5. [4] Write $\vec{w}$ as a linear combination of $\vec{u}$ and $\vec{v}$ where

$$\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \vec{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{ and } \vec{w} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

We need to find scalars c+d so that

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} = a\begin{bmatrix} -1 \\ 2 \end{bmatrix} + b\begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} 1 = -a + b \\ -1 = 2a + 0 \end{matrix}$$

looking for lin combo (+1)

So we have a lin. sys. to solve for

$$\left[\begin{array}{cc|c} -1 & 1 & 1 \\ 2 & 0 & -1 \end{array}\right] \xrightarrow{-R_1} \left[\begin{array}{cc|c} 1 & -1 & -1 \\ 2 & 0 & -1 \end{array}\right] \xrightarrow{R_2 - 2R_1 = NR_2} \left[\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 2 & 1 \end{array}\right]$$

set up equations / work to get it (+1)

$$\xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 1 & \frac{1}{2} \end{array}\right] \xrightarrow{R_1 + R_2 = NR_1} \left[\begin{array}{cc|c} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{1}{2} \end{array}\right]$$

So

$$\vec{w} = -\tfrac{1}{2}\vec{u} + \tfrac{1}{2}\vec{v}$$

right answer / checked work (+1)

Check

$$-\tfrac{1}{2}\begin{bmatrix} -1 \\ 2 \end{bmatrix} + \tfrac{1}{2}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} + \frac{1}{2} \\ -1 + 0 \end{bmatrix} = \begin{bmatrix} +1 \\ -1 \end{bmatrix} \checkmark$$

6. [4] Let $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$, and $X = \cancel{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}$. Given that $AB - 5X = 0$, find X.

$$X = \begin{bmatrix} x_1 & 0 \\ 0 & x_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} - 5\begin{bmatrix} x_1 & 0 \\ 0 & x_2 \end{bmatrix} = 0$$

mult right +1
scaled right +1
add sub right +1
right answer +1

$$\begin{bmatrix} 1+4 & 0 \\ 0 & 1+4 \end{bmatrix} = 5\begin{bmatrix} x_1 & 0 \\ 0 & x_2 \end{bmatrix}$$

$$\frac{1}{5}\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} x_1 & 0 \\ 0 & x_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} x_1 & 0 \\ 0 & x_2 \end{bmatrix}$$

$$x_1 = x_2 = 1$$