MIDTERM # Math 341 Summer 2008

NAME:

True/False: If the statement is *always* true, give a brief explanation of why it is. If the statement is false, give a counterexample. Let A and B be matrices, and c be a scalar.

1. [4] The matrices $\begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 0 \end{bmatrix}$ and $\begin{bmatrix} 2 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$ are equal.

2. [4]Every matrix can be reduced to one and only one matrix in reduced row form.

3. [4] If $cA = 0$, then either $c = 0$ or $A = 0$.

4. [4] The matrix $\begin{bmatrix} 2 & 1 \\ 3 & -2 \end{bmatrix}$ is invertible.

5. [4]The function $f : \mathbb{R} \to \mathbb{R}$ defined by the rule $f(x) = x + 1$ is a linear function.

6. [4] If a matrix A is diagonal then A is not upper triangular.

Free Responce: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

7. [6] Complete the following definition: A map $T : \mathbb{R}^n \to \mathbb{R}^m$ is a *linear* transformation if...

8. [10] Let $A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 2 & 4 & 8 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

   (a) Find the reduced row echelon form of A.

   (b) What is the rank and nullity of A?

   (c) Use Gaussian elimination to find the solution set to the following system of linear equations: $\left\{ \begin{array}{c} x_1 + x_2 + x_3 = 2 \\ 2x_1 + 2x_2 + 4x_3 = 8 \\ x_3 = 2 \end{array} \right\}$

9. [20] For the following problem let

$$A = \begin{bmatrix} 1 & 1 \\ 3 & -2 \end{bmatrix} \; B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \; C = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$$

Perform the following calculations where possible:

(a) $3A - B$

(b) AB

(c) BA

(d) Find the inverse of C if it exists.

(e) Find the determinant of C.

10. [15] Let $T_A : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear operator defined by the rule $T_A\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = A\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ where A is some matrix and A acts on the vector by standard matrix multiplication.

(a) What are the dimensions of A?

(b) Given that $T_A\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $T_A\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$, and $T_A\left(\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$, find the matrix A that corresponds to the function T_A.

11. [15] Let V be all diagonal 2×2 matrices with real entries. That is let $V = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} | a, b \in \mathbb{R} \right\}$ with the standard matrix addition and scalar multiplication. Determine if V is a vector space. Yes, I really do want you to run through the 10 properties.