§2.6 #1~~cd~~, 3~~de~~, 4~~ab~~, 9~~bc~~, 10~~bc~~, 11~~bc~~, ~~19~~, ~~21~~

1 c) $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$ $A_{11}=+(0)=0$, $A_{12}=-0=0$, $A_{13}=0$

$A_{21}=-(0)=0$, $A_{22}=+(4)=4$, $A_{23}=-0$

$A_{31}=+(0)=0$, $A_{32}=-0=0$, $A_{33}=0$

so $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} + & - \\ - & + \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 7+i & 1 \end{bmatrix}$

3) d) $\begin{vmatrix} 1 & -1 & 4 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 2 & 7 \\ 2 & 3 & 4 & 6 \end{vmatrix} = 1\begin{vmatrix} 1 & 0 & 3 \\ 0 & 2 & 7 \\ 3 & 4 & 6 \end{vmatrix} - \cancel{0 \cdot 1} + \cancel{0 \cdot 1} - 2\begin{vmatrix} -1 & 4 & 2 \\ 1 & 0 & 3 \\ 0 & 2 & 7 \end{vmatrix}$

3d

-10

$= 1\begin{vmatrix} 1 & 0 & 3 \\ 0 & 2 & 7 \\ 0 & 4 & -3 \end{vmatrix} - 2\begin{vmatrix} -1 & 4 & 2 \\ 0 & 4 & 5 \\ 0 & 2 & 7 \end{vmatrix} = 1\left(1\cdot\begin{vmatrix} 2 & 7 \\ 4 & -3 \end{vmatrix}\right) - 2\left(-1\begin{vmatrix} 4 & 5 \\ 2 & 7 \end{vmatrix}\right)$

R_3-3R_1 $R_1+R_2=NR_1$

$=(-6+28)+2(14-10)=22+2\cdot4=22+8=30$ is inv

e) $\begin{vmatrix} -1 & -1 \\ 1 & 1-2i \end{vmatrix} = (-1)(1-2i)-(1)(-1) = -1+2i+1 = 2i$ is inv

Check: 3d 4ab

4a) $(1)(1)(2)(6)=12\neq 0$
invertible

b) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$ $-1\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = -1$ is inv.

9) $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = A$ $M(A) = \begin{vmatrix} \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} & \begin{vmatrix} 0 & 0 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & 3 \\ 0 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & 3 \\ 1 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 0 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \end{vmatrix}$

$= \begin{bmatrix} -1 & 0 & -1 \\ 0 & -4 & 0 \\ -3 & 0 & 1 \end{bmatrix}$

$adj(A) = \begin{bmatrix} -1 & 0 & -3 \\ 0 & -4 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$A \cdot adj(A) = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -4 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{bmatrix}$ ✓

c) $\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} = A \quad M(A) = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \quad adj(A) = \begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$

$A \cdot adj(A) = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$ ✓

10b) $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} = A \quad M(A) = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad adj(A) = \begin{bmatrix} -2 & -1 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$M(A)\,adj(A) = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} -2 & -1 & 0 \\ -1 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 1+i \\ 1-i & 2 \end{bmatrix} = A \quad M(A) = \begin{bmatrix} 2 & 1-i \\ 1+i & 1 \end{bmatrix} \quad adj(A) = \begin{bmatrix} 2 & -1-i \\ -1+i & 1 \end{bmatrix}$

$(1+i)(-1+i) = -1 + \cancel{i - i} - 1$

$M(A)\,adj(A) = \begin{bmatrix} 1 & 1+i \\ 1-i & 2 \end{bmatrix}\begin{bmatrix} 2 & -1-i \\ -1+i & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

note $\det(A) = 1(2) - (1+i)(1-i) = 2 - (1 \cancel{-i+i} + 1) = 2-2 = 0$

11b) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix} = A \quad M(A) = \begin{bmatrix} 2 & 1 & -2 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \quad adj(A) = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & -1 \\ -2 & 0 & 2 \end{bmatrix}$

$\det(A) = 1(2-0) = 2$ and $A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 0 & 0 \\ -1 & 1 & -1 \\ -2 & 0 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 1 & -2 & 2 \\ -1 & 2 & -1 \\ 1 & -3 & 1 \end{bmatrix} = A \quad M(A) = \begin{bmatrix} -1 & 0 & 1 \\ 4 & -1 & -1 \\ -2 & 1 & 0 \end{bmatrix} \quad adj(A) \begin{bmatrix} -1 & -4 & -2 \\ 0 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$

$\det(A) = \begin{vmatrix} 1 & -2 & 2 \\ 0 & 0 & 1 \\ 0 & -1 & -1 \end{vmatrix} = -\begin{vmatrix} 1 & -2 & 2 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{vmatrix} = 1$

$A^{-1} = \begin{bmatrix} -1 & -4 & -2 \\ 0 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$

19) Show that if A is invertible then $\det(A)\cdot\det(A^{-1})=1$.

Since A is invertible A^{-1} exists and $AA^{-1}=I_n$ for a suitable size identity matrix.
Recall that the det. is multiplicative
ie Prop D6: $\det AB = \det A \cdot \det B$
Thus $\det(A)\cdot\det(A^{-1}) = \det(A\cdot A^{-1})$ by the recollection
$= \det(I)$ b/c A^{-1} is the inv. of A
Since I is an upper triangular matrix the determinant is the product of the entries on the diagonal. All the entries on the diagonal are 1, $\det(I)=1$.
Thus $\det(A)\cdot\det(A^{-1})=1$.

21) If A is a 5x5 matrix, figure out how to write $\det(-2A)$ in terms of $\det(A)$.

Notice that $-2A=[-2a_{ij}]$ where $A=[a_{ij}]$. In particular the 1st row of the matrix $-2A$ is the same as the 1st row of A but with a scalar multiple. The same is true for the 2nd row of $-2A$ & A. In fact the same is true for the 3rd, 4th & 5th rows.
The matrix $-2A$ can in fact be obtained from the matrix A by mult each row by -2. This is an elementary row operation (cR_i) performed 5 times so we can use D2 to write $\det(-2A) = (-2)^5 \det(A)$

The same reasoning works for any scalar so $\det(cA) = c^5 \det(A)$