only need the elementary matrices themselves.

§2.5 #2, 3, ~~7~~, 8, 15, 18, 20, 21

tried all miss up to 2 (+2) (+2)

2) a) $\left[\begin{array}{ccc|ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 4 & 10 & 0 & 1 & 0 \\ 9 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \Longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/8 & 0 & 1/8 \\ 0 & 1 & 0 & 3/8 & 0 & -1/24 \\ 0 & 0 & 1 & -3/20 & 1/10 & 1/60 \end{array}\right]$

b) $\left[\begin{array}{ccc|ccc} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{array}\right]$

c) $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{array}\right]$

d) $\left[\begin{array}{cc|cc} 1 & a & 1 & 0 \\ a & 1 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cc|cc} 1 & a & 1 & 0 \\ 0 & 1-a^2 & -a & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & a & 1 & 0 \\ 0 & 1 & \frac{-a}{1-a^2} & \frac{1}{1-a^2} \end{array}\right]$

if $a \neq 0$

if $a=0$ is its own inverse.

$\rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1+\frac{a^2}{1-a^2} & -\frac{a}{1-a^2} \\ 0 & 1 & \frac{-a}{1-a^2} & \frac{1}{1-a^2} \end{array}\right]$

note $1+\frac{a^2}{1-a^2} = \frac{1}{1-a^2}$ thus is also $\frac{1}{1-a^2}\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix}$ only if $a \neq 1$ or -1

e) $\left[\begin{array}{cc|cc} i+1 & 0 & 1 & 0 \\ 1 & i & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{2}-\frac{1}{2}i & 0 \\ 1 & i & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{2}-\frac{1}{2}i & 0 \\ 0 & i & -\frac{1}{2}+\frac{1}{2}i & 1 \end{array}\right]$

$\rightarrow \left[\begin{array}{cc|cc} 1 & 0 & \frac{1}{2}-\frac{1}{2}i & 0 \\ 0 & 1 & \frac{1}{2}+\frac{1}{2}i & -i \end{array}\right]$

$\frac{1}{1+i}\frac{(1-i)}{(1-i)}$ $\frac{1-i}{1+1}$

$\frac{1}{i}\frac{(-i)}{(-i)} = +i$

$(-\frac{1}{2}+\frac{1}{2}i)(-i)$

$-\frac{1}{2}-\frac{1}{2}i$

$-i/(-i)$

Check $\begin{bmatrix} i+1 & 0 \\ 1 & i \end{bmatrix}\begin{bmatrix} \frac{1}{2}-\frac{1}{2}i & 0 \\ -\frac{1}{2}+\frac{1}{2}i-1 & \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3) a) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$ $\quad \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ formula

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 7 \\ -2 \end{bmatrix} = \begin{bmatrix} 20 \\ -11 \end{bmatrix}$ Check $40 + -33 = 7$ ✓

$20 + 22 = 2$ ✓

b) $\begin{bmatrix} 3 & 6 & -1 \\ -2 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$ need the inverse

$\left[\begin{array}{ccc|ccc} 3 & 6 & -1 & 1 & 0 & 0 \\ -2 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{3}R_1} \left[\begin{array}{ccc|ccc} 1 & 2 & -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ -2 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2+2R_1 = NR_2} \left[\begin{array}{ccc|ccc} 1 & 2 & -\frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 5 & \frac{1}{3} & \frac{2}{3} & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

.... $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 1 & -6 & 7 \\ 2 & 3 & -1 \\ 0 & 0 & 15 \end{bmatrix}\begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ ✓

c) $\begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}^{-1} = \frac{-1}{3}\begin{bmatrix} 2 & -1 \\ -5 & 1 \end{bmatrix}$ so

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{-1}{3}\begin{bmatrix} 2 & -1 \\ -5 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

5) a) elimination $R_2 - 3R_1 = NR_2$ so $\underbrace{\begin{bmatrix}1&0&0\\-3&1&0\\0&0&1\end{bmatrix}}_{\text{elementary matrix \& inverse to}}\begin{bmatrix}1&0&0\\3&1&0\\0&0&1\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

book notation: $E_{21}(-3)$

b) scaling R_2 by $-\frac{1}{2}$ $\quad \underbrace{\begin{bmatrix}1&0\\0&-\frac{1}{2}\end{bmatrix}}_{\text{elem. matrix \& inverse to}}\begin{bmatrix}1&0\\0&-2\end{bmatrix} = I_2$

book notation: $E_2(-\frac{1}{2})$

c) Swap $R_1 \leftrightarrow R_3$ then $\quad \begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}\begin{bmatrix}0&0&1\\1&1&0\\1&0&0\end{bmatrix}$

better way to record this: one elem. matrix

$$\left[\begin{array}{ccc|ccc}0&0&1&1&0&0\\1&1&0&0&1&0\\1&0&0&0&0&1\end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|ccc}1&0&0&0&0&1\\1&1&0&0&1&0\\0&0&1&1&0&0\end{array}\right]$$

$$\left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\1&1&0&0&1&0\\0&0&1&0&0&1\end{array}\right] \xrightarrow{R_2 - R_1 = NR_2} \left[\begin{array}{ccc|ccc}1&0&0&1&0&0\\0&1&0&-1&1&0\\0&0&1&0&0&1\end{array}\right]$$

so the inverse is: $\begin{bmatrix}1&0&0\\-1&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$

book notation: $E_{21}(-1)\,E_{13}$

d) $$\left[\begin{array}{ccc|ccc}1&-1&0&1&0&0\\0&1&-1&0&1&0\\0&0&1&0&0&1\end{array}\right] \xrightarrow{R_2 + R_3 = NR_2} \left[\begin{array}{ccc|ccc}1&-1&0&1&0&0\\0&1&0&0&1&1\\0&0&1&0&0&1\end{array}\right]$$

$$\left[\begin{array}{ccc|ccc}1&-1&0&1&0&0\\0&1&0&0&1&0\\0&0&1&0&0&1\end{array}\right] \xrightarrow{R_1 + R_2 = NR_1} \left[\begin{array}{ccc|ccc}1&0&0&1&1&0\\0&1&0&0&1&0\\0&0&1&0&0&1\end{array}\right]$$

so the inverse written as elem. matrices: $\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&1\\0&0&1\end{bmatrix}$

book notation: $E_{12}(1)\,E_{23}(1)$

e) $$\left[\begin{array}{cc|cc}-1&0&1&0\\i&3&0&1\end{array}\right] \xrightarrow{-R_1} \left[\begin{array}{cc|cc}1&0&-1&0\\i&3&0&1\end{array}\right]$$

$$\left[\begin{array}{cc|cc}1&0&1&0\\i&3&0&1\end{array}\right] \xrightarrow{R_2 - iR_1 = NR_2} \left[\begin{array}{cc|cc}1&0&1&0\\0&3&-i&1\end{array}\right]$$

$$\left[\begin{array}{cc|cc}1&0&1&0\\0&3&0&1\end{array}\right] \xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{cc|cc}1&0&1&0\\0&1&0&\frac{1}{3}\end{array}\right]$$

so the inverse written as elem. matrices: $\begin{bmatrix}1&0\\0&\frac{1}{3}\end{bmatrix}\begin{bmatrix}1&0\\-i&1\end{bmatrix}\begin{bmatrix}-1&0\\0&1\end{bmatrix}$

book notation $E_2(\frac{1}{3})\,E_{21}(-i)\,E_1(-1)$

Stoke (+1) Curb (+1) gal,1 (+1) R to 1

8) $AX = B$ where $A = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & -1 & 1 & 1 \end{bmatrix}$

$X = A^{-1}B$

if A^{-1} exists... $\frac{1}{5-4}\begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix} = A^{-1}$

$$X = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 7 & -2 & -12 \\ 0 & -3 & 1 & 5 \end{bmatrix}$$

13) I'm going to thm 2.7

a) $A = \begin{bmatrix} 1 & k \\ 0 & -1 \end{bmatrix}$ this is in reduced row form,
Rank is the # of nonzero rows in rrf (pg 39)
So $rk\,A = 2$ so thm 2.7 part 6 $\Rightarrow A$
is invertible no matter what k is.

$$A^{-1} = \frac{1}{-1 \cdot 0}\begin{bmatrix} -1 & -k \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -k \\ 0 & -1 \end{bmatrix}$$

b) $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ k & 0 & 1 \end{bmatrix}$ Again using Thm 2.7 B is invertible
if the $rk\,B = 3$. Recall that ero
do not affect the rank so

$$rk\,B = rk\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1-k \end{pmatrix} = \begin{cases} 2 & \text{if } 1-k=0 \\ 3 & \text{else} \end{cases}$$

so B has an inverse as long as $1-k \neq 0$ or $k \neq 1$

finding the inverse

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ k & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_3 - kR_1 = NR_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1-k & -k & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{1-k}R_3}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{-k}{1-k} & 0 & \frac{1}{1-k} \end{array}\right] \xrightarrow[R_1 - R_3 = NR_3]{R_2 - R_3 = NR_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1+\frac{k}{1-k} & 0 & \frac{-1}{1-k} \\ 0 & 1 & 0 & \frac{k}{1-k} & 1 & \frac{-1}{1-k} \\ 0 & 0 & 1 & \frac{k}{1-k} & 0 & \frac{1}{1-k} \end{array}\right]$$

inverse $\frac{1}{1-k}\begin{bmatrix} 1 & 0 & -1 \\ k & 1-k & -1 \\ -k & 0 & 1 \end{bmatrix}$

c) $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & 0 & k \end{bmatrix}$ Again by thm 2.7 C is invertible if $rk\, C = 4$

So C is invertible as long as $k \neq 0$

$C^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1/6 & 0 \\ 0 & 0 & 0 & 1/k \end{bmatrix}$

T/F → 15) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

rk is 2 $\Downarrow$ invertible | rk is 2 $\Downarrow$ invertible | rk is 0 $\Downarrow$ not invertible.

16) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Stated (1)

rk is 1 $\Downarrow$ singular | rk is 1 $\Downarrow$ singular | rk is 2 $\Downarrow$ non singular

21) Show A that has the prop $A^3 - 2A + 3I = 0$ is invertible.

$A^3 - 2A + 3I = 0$
$\Rightarrow A^3 - 2A = -3I$ by using the additive inverse
$\Rightarrow -\frac{1}{3}(A^3 - 2A) = I$ by using assoc & scalars.
$\Rightarrow -\frac{1}{3}A^3 - \frac{2}{3}A = I$ by dist.
$\Rightarrow (-\frac{1}{3}A^2 - \frac{2}{3}I)A = I$ by dist again

So $(-\frac{1}{3}A^2 - \frac{2}{3}I)$ is a left inverse.
We could have used assoc to show also
$A(-\frac{1}{3}A^3) - A\frac{2}{3} = I$
$\Rightarrow A(-\frac{1}{3}A^{3} - \frac{2}{3}I) = I$ so this is a 2-sided inverse.