§2.4 #1, 3, 5a/b, 6a/b, 10, 14, 21, 23

1) a) $\begin{bmatrix}1&0&0\\0&1&3\\0&0&1\end{bmatrix}$ b) $\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$ c) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&2\end{bmatrix}$ d) $E_{23}^T(-1)=(E_{23}(-1))^T$ $\begin{bmatrix}1&0&0\\0&1&0\\0&-1&1\end{bmatrix}$

e) $E_{12}(3)$ $R_1+3R_2=NR_1$ f) $E_{31}(-a)$ $R_3-aR_1=NR_3$ g) $E_2(3)$ $3R_2$ h) $E_{31}(2)$ $R_3+2R_1=NR_3$

3)

5 a) $$\begin{bmatrix}1&2\\1&3\end{bmatrix}\xrightarrow[R_2-R_1=NR_2]{}\begin{bmatrix}1&2\\0&1\end{bmatrix}\xrightarrow[R_1-2R_2=NR_1]{}\begin{bmatrix}1&0\\0&1\end{bmatrix}$$
$$\begin{bmatrix}1&0\\-1&1\end{bmatrix}\qquad\begin{bmatrix}1&-2\\0&1\end{bmatrix}$$

So $$\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}1&-2\\0&1\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}=\begin{bmatrix}1&-2\\0&1\end{bmatrix}\begin{bmatrix}1&0\\-1&1\end{bmatrix}\begin{bmatrix}1&2\\1&3\end{bmatrix}$$

b) $$\begin{bmatrix}1&1&0\\0&1&1\\0&2&2\end{bmatrix}\xrightarrow{R_3-2R_1=NR_3}\begin{bmatrix}1&1&0\\0&1&1\\0&0&0\end{bmatrix}\xrightarrow{R_1-R_2=NR_1}\begin{bmatrix}1&0&-1\\0&1&1\\0&0&0\end{bmatrix}$$
$$\begin{bmatrix}1&0&0\\0&1&0\\-2&0&1\end{bmatrix}\qquad\begin{bmatrix}1&-1&0\\0&1&0\\0&0&1\end{bmatrix}$$

So $$\begin{bmatrix}1&0&-1\\0&1&1\\0&0&0\end{bmatrix}=\begin{bmatrix}1&-1&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&1&0\\0&1&1\\0&0&0\end{bmatrix}=\begin{bmatrix}1&-1&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\-2&0&1\end{bmatrix}\begin{bmatrix}1&1&0\\0&1&1\\0&2&2\end{bmatrix}$$

6) a) $$\underset{(3\times2)}{\begin{bmatrix}2&1\\0&1\\0&2\end{bmatrix}}\xrightarrow{R_3-2R_2=NR_3}\begin{bmatrix}2&1\\0&1\\0&0\end{bmatrix}\xrightarrow{\frac12R_1}\begin{bmatrix}1&\frac12\\0&1\\0&0\end{bmatrix}\xrightarrow{R_1-\frac12R_2=NR_1}\begin{bmatrix}1&0\\0&1\\0&0\end{bmatrix}$$
$$\underset{(3\times3)}{\begin{bmatrix}1&0&0\\0&1&0\\0&-2&1\end{bmatrix}}\qquad\begin{bmatrix}\frac12&0&0\\0&1&0\\0&0&1\end{bmatrix}\qquad\begin{bmatrix}1&-\frac12&0\\0&1&0\\0&0&1\end{bmatrix}$$

So $$\begin{bmatrix}1&0\\0&1\\0&0\end{bmatrix}=\begin{bmatrix}1&-\frac12&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}\frac12&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&1&0\\0&-2&1\end{bmatrix}\begin{bmatrix}2&1\\0&1\\0&2\end{bmatrix}$$

calc errors 2x -1
Started (+1)
Sec bit on parts

b) $$\begin{bmatrix}1&1&1\\0&0&0\\1&2&2\end{bmatrix}\xrightarrow{R_2\leftrightarrow R_3}\begin{bmatrix}1&1&1\\1&2&2\\0&0&0\end{bmatrix}\xrightarrow{R_2-R_1=NR_2}\begin{bmatrix}1&1&1\\0&1&1\\0&0&0\end{bmatrix}\xrightarrow{R_1-R_2}\begin{bmatrix}1&0&0\\0&1&1\\0&0&0\end{bmatrix}$$
$$\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\qquad\begin{bmatrix}1&0&0\\-1&1&0\\0&0&1\end{bmatrix}\qquad\begin{bmatrix}1&-1&0\\0&1&0\\0&0&1\end{bmatrix}$$

So $$\begin{bmatrix}1&0&0\\0&1&1\\0&0&0\end{bmatrix}=\begin{bmatrix}1&-1&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\-1&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\begin{bmatrix}1&1&1\\0&0&0\\1&2&2\end{bmatrix}$$

wrong size -1

(10) $AB = \begin{bmatrix} R & O \\ S & T \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix} = \left[\begin{array}{ccc|cc} 1 & 1 & 0 & 0 & 0 \\ \hline 1 & 1 & 1 & 1 & -1 \\ 1 & 2 & 1 & 2 & 2 \end{array}\right] \left[\begin{array}{cc} 1 & 0 \\ 1 & 2 \\ 1 & 1 \\ \hline 3 & 1 \\ 0 & 1 \end{array}\right]$

$= \begin{bmatrix} RU + O \\ \hline SU + TV \end{bmatrix}$

(3× 5) (5×2) (3×2)

$= \begin{bmatrix} 2 & 2 \\ 6 & 3 \\ 10 & 9 \end{bmatrix}$

$RU = [1\ 1\ 0] \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 1 \end{bmatrix} = [2\ 2]$ 1×2

$TV = \begin{bmatrix} 1 & -1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 6 & 4 \end{bmatrix}$

$SU = \underset{2\times3}{\overset{1\times3}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}}} \underset{(3\times2)}{\overset{3\times2}{\begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}}} = \begin{bmatrix} 3 & 3 \\ 4 & 5 \end{bmatrix}$

(+) up to line 3 wrong

T/F → (14) a) sym & hermitian

b) not sym or hermitian
(the transpose is a different sized matrix?)

c) $\begin{bmatrix} i & 1 \\ -1 & i \end{bmatrix}^T = \begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix}$ not sym or hermitian.

d) sym & hermitian.

T/F → 21) Let A be an upper Δ matrix that is also symmetric.
Since A is upper Δ, $a_{ij} = 0$ if $i > j$.
Since A is symmetric, $A = A^T \Rightarrow a_{ij} = a_{ji}$.
Thus $a_{ij} = 0$ if $i > j$ also implies $a_{ij} = 0$ if $j < i$
All the entries off the diagonal are thus zero
$\Rightarrow$ A is diagonal.
A symmetric argument works if A is lower Δ.

23) Let $\{E_1, \dots E_n\}$ & $\{F_1, \dots F_m\}$ be elementary row op
so that $E_n \cdots E_1 A = R_A$ & $F_m \cdots F_1 B = R_B$ where R_A
and R_B are in rref. We use E_i's on C in the same order
to obtain $C \sim \begin{bmatrix} R_A & 0 \\ 0 & B \end{bmatrix}$. If we temp. label the first row containing