§2.3 ~~1b,6~~, ~~2a,c,d~~, ~~3~~, ~~5~~, ~~7~~ (±1, ±1) rotation

1 a) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ x-axis returns to self

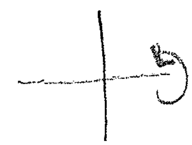
y-axis flips vertically $\begin{bmatrix} 0 \\ y \end{bmatrix} \xrightarrow{T_A} \begin{bmatrix} 0 \\ -y \end{bmatrix}$

$(1,1)^T \longmapsto (1,-1)^T$

$(1,-1)^T \longmapsto (1,1)^T$

$(-1,1)^T \longmapsto (-1,-1)^T$

$(-1,-1)^T \longmapsto (-1,1)^T$



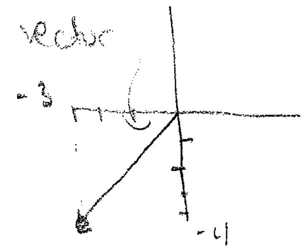
b) $\frac{1}{5}\begin{bmatrix} -3 & -4 \\ -4 & 3 \end{bmatrix}$

x-axis moves onto vector

$\begin{bmatrix} x \\ 0 \end{bmatrix} \longmapsto \begin{bmatrix} -3x \\ -4x \end{bmatrix}$

y-axis

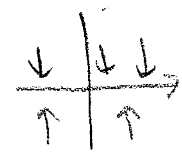
$\begin{bmatrix} 0 \\ y \end{bmatrix} \longmapsto \begin{bmatrix} -4y \\ 3y \end{bmatrix}$



2 a) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

x-axis returns to self

y-axis flattens



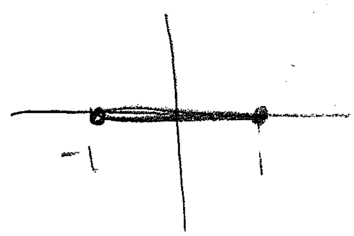
$\begin{bmatrix} 0 \\ y \end{bmatrix} \longmapsto \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$(1,1)^T \longmapsto (1,0)^T$

$(1,-1)^T \longmapsto (1,0)^T$

$(-1,1)^T \longmapsto (-1,0)^T$

$(-1,-1)^T \longmapsto (-1,0)^T$



c) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\begin{bmatrix} x \\ 0 \end{bmatrix} \longmapsto \begin{bmatrix} 1x \\ 1x \end{bmatrix}$ rotates $\pi/4$

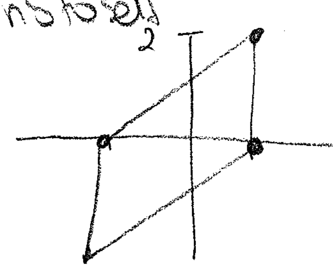
$\begin{bmatrix} 0 \\ y \end{bmatrix} \longmapsto \begin{bmatrix} 0 \\ y \end{bmatrix}$ returns to self

$(1,1)^T \longmapsto (1,2)^T$

$(1,-1)^T \longmapsto (1,0)^T$

$(-1,1)^T \longmapsto (-1,0)^T$

$(-1,-1)^T \longmapsto (-1,-2)^T$



d) $\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix}$

$\begin{bmatrix} x \\ 0 \end{bmatrix} \longmapsto \begin{bmatrix} 2x \\ 3x \end{bmatrix}$

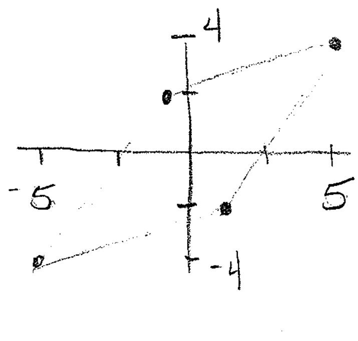
$\begin{bmatrix} 0 \\ y \end{bmatrix} \longmapsto \begin{bmatrix} 3y \\ y \end{bmatrix}$

$(1,1)^T \longmapsto (5,4)^T$

$(1,-1)^T \longmapsto (-1,2)^T$

$(-1,1)^T \longmapsto (1,-2)^T$

$(-1,-1)^T \longmapsto (-5,-4)^T$



3) a) $\begin{bmatrix} 1 & 1 \\ 2 & 0 \\ -1 & 4 \end{bmatrix}$ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ ✓ (3×2)

back is wrong (switched but)

b) not a linear sys.

c) $\begin{bmatrix} 0 & 0 & 2 \\ -1 & 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

5) $x \longmapsto 2x$
$y \longmapsto 4y$ $\Rightarrow \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$

rotation $\begin{bmatrix} \cos \pi/6 & -\sin \pi/6 \\ \sin \pi/6 & \cos \pi/6 \end{bmatrix}$

started (+1)
got it (+1)
second half (1)

scale & then rotate

$$\begin{bmatrix} \cos \pi/6 & -\sin \pi/6 \\ \sin \pi/6 & \cos \pi/6 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 2\cos \pi/6 & -4\sin \pi/6 \\ 2\sin \pi/6 & 4\cos \pi/6 \end{bmatrix} \text{ \underline{etc}}$$

17) Assume $T_A = T_B$ where $T_A : \mathbb{R}^n \longrightarrow \mathbb{R}^m$
and T_B will also corresponds to an $m \times n$ matrix denoted B, $\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \longmapsto A \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$

start (+1)
sense (+1)
get it (+1)

Since $T_A = T_B$ for <u>all</u> vectors $T_A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = T_B \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

But $T_A \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{m1} \end{bmatrix}$ and $T_B \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} b_{11} \\ b_{21} \\ b_{m1} \end{bmatrix}$.

Thus the first column of A = B.
In fact T_A & T_B acting on the vector $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ & the j^{th} row

will show the j^{th} col of A equals the j^{th} col of B. Thus A=B.