§2.1

1) a) ✓   c) ✓   e) ✓

b) ✓   d) not possible   f) ✓

+1 for each

4) a) $X + 3A = C$

$X = C - 3A$

$= \begin{bmatrix} -2 & 1 & 3 \\ -1 & -2 & -6 \end{bmatrix}$

b) $A - 3X = 3C$

$-3X = 3C - A$

$X = -C + \frac{1}{3}A$

$= \begin{bmatrix} -2/3 & -1 & -1/3 \\ -5/3 & -2/3 & 2/3 \end{bmatrix}$

c) $2X + \begin{bmatrix} 2 & 2 \\ 1 & -2 \end{bmatrix} = B$

note this is really

$2X + B = B$

$\Rightarrow 2X = 0$

$\Rightarrow X = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

5) a) $x\begin{bmatrix} 1 \\ 2 \end{bmatrix} + y\begin{bmatrix} 2 \\ 0 \end{bmatrix} + z\begin{bmatrix} 0 \\ -1 \end{bmatrix}$

b) $x\begin{bmatrix} 1 \\ 2 \end{bmatrix} + y\begin{bmatrix} -1 \\ 3 \end{bmatrix}$

c) $x\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} + y\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + z\begin{bmatrix} 0 \\ -1 \\ 5 \end{bmatrix}$

d) $x\begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix} + y\begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix} + z\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

8) $\begin{bmatrix} d & 2a \\ 2d & a \end{bmatrix} = \begin{bmatrix} a-b & b+c \\ a+b & c-b+1 \end{bmatrix} \Rightarrow$

$d = a - b$ $\qquad a - b - d = 0$

$2a = b + c$ $\quad \Rightarrow \quad 2a - b - c = 0$

$2d = a + b$ $\qquad a + b - 2d = 0$

$a = c - b + 1$ $\qquad a + b - c = 1$

found sys of eq to solve x2

had to solve +1

$$\begin{array}{cccc} a & b & c & d \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & -1 & 0 \\ 2 & -1 & -1 & 0 & 0 \\ 1 & 1 & 0 & -2 & 0 \\ 1 & 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow[R_3 - R_1 = NR_3,\ R_4 - R_1 = NR_4]{R_2 - 2R_1 = NR_2} \left[\begin{array}{cccc|c} 1 & -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 2 & 0 & -1 & 0 \\ 0 & 2 & -1 & 1 & 1 \end{array}\right]$$

$R_3 - 2R_2 = NR_3$

$R_4 - 2R_2 = NR_4$

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 0 & 2 & -5 & 0 \\ 0 & 0 & 1 & -3 & 1 \end{array}\right] \xrightarrow[R_4 - R_3 = NR_4]{\frac{1}{2}R_3} \left[\begin{array}{cccc|c} 1 & -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & -5/2 & 0 \\ 0 & 0 & 0 & -1/2 & 1 \end{array}\right] \xrightarrow[R_3 + 5/2 R_4,\ R_2 - 2R_4,\ R_1 + R_4]{-2R_4} \left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -2 \\ 0 & 1 & -1 & 0 & 4 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

$R_2 + R_3 = NR_2$

$R_1 + R_2 = NR_1$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

So $a = -3$ $\quad b = -1$

$c = -5$ $\quad d = -2$

Check ✓ ✓ ✓ ✓

(16) $\begin{bmatrix} 3 & 3 \\ 1 & -3 \end{bmatrix} = a\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + c\begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$

for unknown scalars a, b & c

$3 = a + 0b + 0c$
$3 = a + b + 2c$
$1 = a + b$
$-3 = 0a + b - c$

$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 1 & 1 & 2 & 3 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & -3 \end{array}\right] \xrightarrow[R_3 - R_1 = NR_3]{R_2 - R_1 = NR_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 1 & -1 & -3 \end{array}\right]$

$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & -1 & -3 \end{array}\right] \xrightarrow{\substack{R_3 - R_2 = NR_3 \\ R_4 - R_2 = NR_4}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & -1 & -1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$a = 3 \quad b = -2 \quad c = 1$

Check $3 + -2 + 2(1) = 3$ ✓
$3 + -2 = 1$ ✓
$-2 - 1 = -3$ ✓

(17) a) Assume $A = -A$. We also denote $A = [a_{ij}]$.
We add A to both sides to get $A + A = -A + A$.
Recall on that we can write $-A = [-a_{ij}]$.
Thus we have

$[a_{ij}] + [a_{ij}] = [-a_{ij}] + [a_{ij}]$

$\Rightarrow [a_{ij} + a_{ij}] = [-a_{ij} + a_{ij}]$ by definition of matrix addition.

So $[2a_{ij}] = [0]$.

Since our entries $a_{ij} \in \mathbb{R}$ or $\mathbb{C}$ $\Rightarrow a_{ij} = 0$.
Thus $A = 0$.

b) Let A be a $m \times n$ matrix & c a scaler so that $cA = 0$.
The entry in the i^{th} row & j^{th} col of cA is ca_{ij}.
Since $cA = 0$ we know $ca_{ij} = 0$ for $1 \le i \le m$ and $1 \le j \le n$. Using the zero property of $\mathbb{R}$ or $\mathbb{C}$ this means for each entry either $c = 0$ or $a_{ij} = 0$.

If $c = 0$ then we are done.
Assume $c \neq 0$. The above states $a_{ij} = 0$ for <u>all</u> $1 \le i \le m$ and $1 \le j \le n$. Thus all the entries in A are $0 \Rightarrow A = 0$.

c) Assume $B = cB$.
Add $-B$ to both sides to obtain $B - B = cB - B$.
By definition of $-B$ we can simplify to $0 = cB - B$.
The distributive property implies $0 = (c-1)B$
Since $c \neq 1$, $c - 1 \neq 0$
so part b implies $B = 0$.