§1.4 1, 3, 4, 5, 7, 10, 17, 18 Partial answers

1) a) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

	a)	b)	c)	d)
rr:	yup	yup	nope	yup
rre:	yup	nope	nope	nope

e) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ f) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ g) $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ h) $[1, 3]$

	e)	f)	g)	h)
rr:	yup	yup	nope	yup
rre:	yup	yup	nope	yup

3) a) $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ b) $\begin{bmatrix} \bar{0} \end{bmatrix}$ c) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}$ e) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

	a)	b)	c)	d)	e)
rk:	3	0	3	1	1

missed 2/3
mis 3 1/3

/3 4) a) 2 b) 1 c) 3 d) 0

5) a) $\begin{bmatrix} 1 & -1 & 2 \\ 1 & 3 & 4 \\ 2 & 2 & 6 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 5/2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}$ rk 2

b) $\begin{bmatrix} 3 & 1 & 9 & 2 \\ -3 & 0 & 6 & -5 \\ 0 & 0 & 1 & 2 \end{bmatrix} \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 17/3 \\ 0 & 1 & 0 & -33 \\ 0 & 0 & 1 & 2 \end{array}\right]$ rk 3

c) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ rk 2

7) perform ero on

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & -1 & 2 \\ 2 & 1 & 0 & -2 & 1 \\ 2 & 2 & 2 & -2 & 4 \end{array}\right] \quad \& \quad \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ -2 & -4 & 0 & 0 & 0 \\ 3 & 6 & -1 & 1 & 0 \end{bmatrix}$$

? ?

10)

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & \\ a_{31} & a_{32} & & \\ a_{41} & a_{42} & \cdots & \end{bmatrix}}_{A} \underbrace{\quad}_{\text{rhs vector}}$$

($4 \times n$ matrix)

in future: ask to justify.

Let $\tilde{A}$ be the augmented matrix with coef matrix A & a rhs vector that has a unique solution.
The contrapositive of thm 1.5 2 $\Rightarrow n \le rk A$.

tried to justify but stated cdf 1

Recall $rk A \le \min\{4,n\}$. So we have $n \le rk A \le \min\{4,n\}$
If $4 < n$:

$$4 < n \le rk\ A \le 4 = \min\{4,n\} \quad \times$$

If $n \le 4$

$$n \le rk\ A \le n = \min\{4,n\} \quad \text{consistent}$$

note it is possible to have $rk A = 4$ ex

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & * \\ 0 & 1 & 0 & 0 & * \\ 0 & 0 & 1 & 0 & * \\ 0 & 0 & 0 & 1 & * \end{array}\right]$$

$rk A \le 4$

Note if $rk\ A = 0$ then the sys. of lin. eq could have either no solutions or infinite. Thus
$1 \le rk A \le 4$

ex $\left[\begin{array}{c|c} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}\right]$ has a ? sol.

17) a) false

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

rk of aug. matrix $= 2 \leq \#$ unknowns $= 3$

b) true; the zero vect. is always a solution

2/3

miss 1.3 answers/correct

c) false; $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{array}\right] \Rightarrow$ inconsistent so no solutions

d) false; $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ equivalent to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

reduced form   reduced form

e) false $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow$ has only one sol. $\{(0,0)\}$.

18) Show that a system of linear equations has a ! solution iff every column, except the last, of the rre form of the augmented matrix has a pivot entry in it.

$\Leftarrow$ Assume every col, except the last, of the rre form of the augmented matrix has a pivot entry. The matrix will be of the form $\left[\begin{array}{cccc|c} 1 & 0 & 0 & & b_1 \\ 0 & 1 & 0 & & b_2 \\ 0 & 0 & 1 & & b_3 \\ & & & & \\ & & & 1 & b_m \end{array}\right]$

$\Rightarrow$ there is a ! solution mainly $\{(b_1, b_2, \ldots b_m)\}$.

$\Leftarrow$ Assume there is a ! solution to the system of linear equations & call it $(c_1, c_2, \ldots)$.
Thm 1.3 guarantees that the system reduces to an equivalent and ! reduced row echelon form. We need to show each col. has a pivot entry in it.

Since the sol set is $\{(c_1, c_2, \ldots)\} = \{(x_1, x_2, \ldots) \mid x_i = c_i \ \forall i\}$

we know the reduced row echelon form is

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & \cdots & & c_1 \\ 0 & 1 & 0 & 0 & \cdots & & c_2 \\ 0 & 0 & 1 & 0 & \cdots & & c_3 \\ & & & & & & \vdots \\ 0 & \cdots & & & & 1 & c_m \end{array}\right]$$

Note that each col contains a leading term. //