1.1

partial answers

(4) a) nope nonlinear

b) $x - y + 2z = 0$

$3x - 2y\ 0 = 0$

c) nope nonlinear

~~(6)~~ a) $m=3$ $n=2$

$a_{11}=1$ $a_{12}=-1$ $b_1=1$

$a_{21}=2$ $a_{22}=-1$ $b_2=3$

$a_{31}=1$ $a_{32}=1$ $b_3=3$

(12)
$$\begin{aligned} a_0 + a_1 + a_2 &= 1 \\ a_0 + 2a_1 + 4a_2 &= 1 \\ a_0 + 3a_1 + 9a_2 &= 2 \end{aligned} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 1 \\ 1 & 3 & 9 & 2 \end{bmatrix}$$ b)

If we were using a polynomial $y = a_0 + a_1x + \ldots + a_nx^n$ to interpolate a function f, the collection of data would give us a linear system of n equations.

1.2

$\sqrt{2}$ is a solution to $x^2 - 2 = 0$ b/c $(\sqrt{2})^2 - 2 = 0$

If $\sqrt{2} \in \mathbb{Q}$ then there would exist $a, b \in \mathbb{N}$

so that $\sqrt{2} = a/b$ so $(a/b)^2 - 2 = 0$.

Then $\frac{a^2}{b^2} = 2 \Rightarrow a^2 = 2b^2$

Write $a = 2^i\alpha$ and $b = 2^j\beta$

where α + β are relatively prime to 2.

The above $a^2 = 2b^2$ becomes $(2^i\alpha)^2 = 2(2^j\beta)^2$

or $2^{2i}\alpha^2 = 2 \cdot 2^{2j}\beta^2$ so $2^{2i}\alpha^2 = 2^{2j+1}\beta^2$.

Since 2 is prime no factors of 2 arise from α^2 or β^2 but the equality shows the product of an even # of 2's is the same as the product of an odd # of 2's.

This is cannot be true thus our assumption that $\sqrt{2} \in \mathbb{Q}$ must have been false.

5) a) $(4+2i)-(3-6i) = 4+2i-3+6i = 1+8i$

b) $(2+4i)(3-i) = 6\cancel{-2i+12i}-4i^2 = 6+4+10i = 10+10i$

c) $\frac{2+i}{2-i}\frac{(2+i)}{(2+i)} = \frac{4+2i+2i+i^2}{4-i^2} = \frac{4-1+4i}{4+1} = \frac{3+4i}{5} = \frac{3}{5}+\frac{4}{5}i$

d) $\frac{1-2i}{1+2i}\frac{(1-2i)}{(1-2i)} = \frac{1-2i-2i+4i^2}{1+4} = \frac{-3-4i}{5} = -\frac{3}{5}-\frac{4}{5}i$

e) $\overline{7(6-i)} = \overline{42-7i} = 42+7i$

6) a) $|2+4i| = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$

b) $-7i^2+6i^3 = -7(-1)+6(-1)i = 7-6i$

c) $(3+4i)(7-6i) = 21-18i+28i-24i^2 = 21+24+10i$
$= 45+10i$

d) $\overline{i(1-i)} = \overline{i-i^2} = \overline{i+1} = \overline{1+i} = 1-i$

1.3

5) a) $\left\{\begin{bmatrix}1-t\\t\\-1\end{bmatrix} \middle| t\in\mathbb{R}\right\}$

b) $\left\{\begin{bmatrix}-1-2t\\t\\-2\\3\end{bmatrix} \middle| t\in\mathbb{R}\right\}$

c) $\left\{\begin{bmatrix}3-2t\\-1-t\\t\end{bmatrix} \middle| t\in\mathbb{R}\right\}$

d) $\left\{\begin{bmatrix}1+\frac{2}{3}i\\1-\frac{1}{3}i\end{bmatrix}\right\}$

e) $\left\{\begin{bmatrix}\frac{7}{11}t\\-\frac{2}{11}t\\\frac{6}{11}t\\t\end{bmatrix} \middle| t\in\mathbb{R}\right\}$

7) a) $\left\{\begin{bmatrix}4\\t\\-2\end{bmatrix} \middle| t\in\mathbb{R}\right\}$

b) $\left\{\begin{bmatrix}1\\2\\2\end{bmatrix}\right\}$

c) inconsistent.

d) $\left\{\begin{bmatrix}t\\s\end{bmatrix} \middle| t,s\in\mathbb{R}\right\}$

8) a) $\left\{ \begin{bmatrix} 0 \\ t \\ 2 \end{bmatrix} \middle| \, t \in \mathbb{R} \right\}$ b) $\left\{ \begin{bmatrix} -1 \\ t \\ 2 \end{bmatrix} \middle| \, t \in \mathbb{R} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ -2 \\ t \end{bmatrix} \middle| \, t \in \mathbb{R} \right\}$ d) inconsistent

9) a) $\left[\begin{array}{cc|c} 1 & -1 & b_1 \\ 1 & 2 & b_2 \end{array}\right] \xrightarrow{R_2 - R_1 = NR_2} \left[\begin{array}{cc|c} 1 & -1 & b_1 \\ 0 & 3 & b_2 - b_1 \end{array}\right] \xrightarrow{\frac{1}{3}R_2}$

$\left[\begin{array}{cc|c} 1 & -1 & b_1 \\ 0 & 1 & \frac{b_2 - b_1}{3} \end{array}\right] \xrightarrow{R_1 + R_2 = NR_1} \left[\begin{array}{cc|c} 1 & 0 & b_1 + \frac{b_2 - b_1}{3} \\ 0 & 1 & \frac{b_2 - b_1}{3} \end{array}\right]$

Solution set: $\left\{ \begin{bmatrix} b_1 + \frac{b_2 - b_1}{3} \\ \frac{b_2 - b_1}{3} \end{bmatrix} \right\}$

b) $\left[\begin{array}{cc|c} 1 & -1 & b_1 \\ 2 & -2 & b_2 \end{array}\right] \xrightarrow{R_2 - 2R_1 = NR_2} \begin{bmatrix} 1 & -1 & b_1 \\ 0 & 0 & b_2 - 2b_1 \end{bmatrix}$

inconsistent unless $0 = b_2 - 2b_1$

$\Rightarrow 2b_1 = b_2 \Rightarrow b_1 = \frac{1}{2} b_2$

in which case $x_1 - x_2 = b_1$

and the sol. set is $\left\{ \begin{bmatrix} b_1 + t \\ t \end{bmatrix} \middle| \, t \in \mathbb{R} \right\}$

c) $\left[\begin{array}{cc|c} i & -1 & b_1 \\ 2 & 2 & b_2 \end{array}\right] \xrightarrow{-iR_1} \left[\begin{array}{cc|c} 1 & i & -b_1 i \\ 2 & 2 & b_2 \end{array}\right] \xrightarrow{R_2 - 2R_1 = NR_1}$

$\left[\begin{array}{cc|c} 1 & i & -b_1 i \\ 0 & 2-2i & b_2 + 2b_1 i \end{array}\right] \xrightarrow{\frac{1}{2-2i}R_2} \left[\begin{array}{cc|c} 1 & i & -b_1 i \\ 0 & 1 & \frac{1}{2-2i}(b_2 + 2b_1 i) \end{array}\right]$

$\xrightarrow{R_2 - iR = NR_2} \left[\begin{array}{cc|c} 1 & 0 & -b_1 i - i\left(\frac{1}{2-2i}(b_2 + 2b_1 i)\right) \\ 0 & 1 & \frac{1}{2-2i}(b_2 + 2b_1 i) \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & * \\ 0 & 1 & \heartsuit \end{array}\right]$

lets simplify things

$* = -b_1 i - \frac{1}{2-2i}(b_2 i - 2b_1) = -b_1 i - \frac{(b_2 i - 2b_1)(2+2i)}{4+4}$

$= -b_1 i - \frac{1}{8}(2b_2 i - 2b_2 - 4b_1 - 4b_1 i)$

$= -b_1 i - \frac{1}{4} b_2 i + \frac{1}{4} b_2 + \frac{1}{2} b_1 + \frac{1}{2} b_1 i$

$$= \left(\tfrac{1}{2}b_1 + \tfrac{1}{4}b_2\right) + \left(-\tfrac{1}{2}b_1 - \tfrac{1}{4}b_2\right)i \qquad \checkmark$$

$$\wp = \frac{1}{2-2i}\left(b_2 + 2b_1 i\right)$$
$$= \frac{(b_2 + 2b_1 i)}{(2-2i)}\frac{(2+2i)}{(2+2i)} = \frac{2b_2 + 2b_2 i + 4b_1 i - 4b_1}{4+4}$$
$$= \left(\tfrac{1}{4}b_2 - \tfrac{1}{2}b_1\right) + \left(\tfrac{1}{4}b_2 + \tfrac{1}{2}b_1\right)i$$

so solution set $\left\{ \begin{bmatrix} \left(\tfrac{1}{2}b_1 + \tfrac{1}{4}b_2\right) - \left(\tfrac{1}{2}b_1 + \tfrac{1}{4}b_2\right)i \\ \left(\tfrac{1}{2}b_2 - \tfrac{1}{2}b_1\right) + \left(\tfrac{1}{4}b_2 + \tfrac{1}{4}b_1\right)i \end{bmatrix} \right\}$

13) use $\left[\begin{array}{cc|ccc} 1 & -1 & 1 & 0 & 2 \\ 1 & -2 & 0 & 1 & 3 \end{array}\right]$ to solve all 3 at one time.

$R_2 - R_1 = NR_2$

$\left[\begin{array}{cc|ccc} 1 & -1 & 1 & 0 & 2 \\ 0 & -1 & -1 & 1 & 1 \end{array}\right]$

$-1R_2$

$\left[\begin{array}{cc|ccc} 1 & -1 & 1 & 0 & 2 \\ 0 & 1 & 1 & -1 & -1 \end{array}\right]$

$R_1 + R_2 = NR_1$

$\left[\begin{array}{cc|ccc} 1 & 0 & 2 & -1 & 1 \\ 0 & 1 & 1 & -1 & -1 \end{array}\right]$