Final Practice Partial Solution **Math 341** Summer 2008

Disclaimer: I have not proof read this at all!!

1. Define carefully the following terms:
   (a) Linearly Independent
   (b) Spanning Set
   (c) Basis
   (d) Vector Space
   (e) Subspace
   (f) Linear Transformation/Funtion/Operator
2. Complete the following for each of the given matrices A.
   (a) Find the reduced row echelon form for A.
   (b) Are the columns of A linearly independent?
   (c) Do the columns of A span $\mathbb{R}^m$ where m is the number of rows in A?
   (d) Find the domain and codomain for the linear transformation T_A.
   (e) Is the linear transformation T_A one-to-one?
   (f) Is the linear transformation T_A onto?
   (g) Find a basis for the column space of A
   (h) Find a basis for the null space of A.
   (i) What is the rank of A?
   (j) Find a basis for the span of T_A.
   (k) Find a basis for the kernel of T_A.
   (l) If possible, compute the determinant of A.
   (m) If possible, determine if A is invertible, and then find A^{-1}.
   (n) Is $\vec{0}$ and eigenvector of A?
   (o) If possible, find the characteristic polynomial of A?
   (p) If possible, find the eigenvalues of A?
3. Do the three lines $x_1 - 4x_2 = 1$, $2x_1 - x_2 = -3$, and $-x_1 - 3x_2 = 4$ have at least one common point of intersection? Explain.

   The three planes have one point in common.

4. Suppose a 3× 5 coefficient matrix for a system has three pivot columns. Is the system consistent? Why or why not?

   Yes. The system is consistent because with three pivots, there must be a pivot in the third (bottom) row of the coefficient matrix. The reduced echelon form cannot contain a row of the form [000001].

5. Suppose a system of linear equations has a 3×5 augmented matrix whose fifth column is a pivot column. Is the system consistent? Why or why not?

6. Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

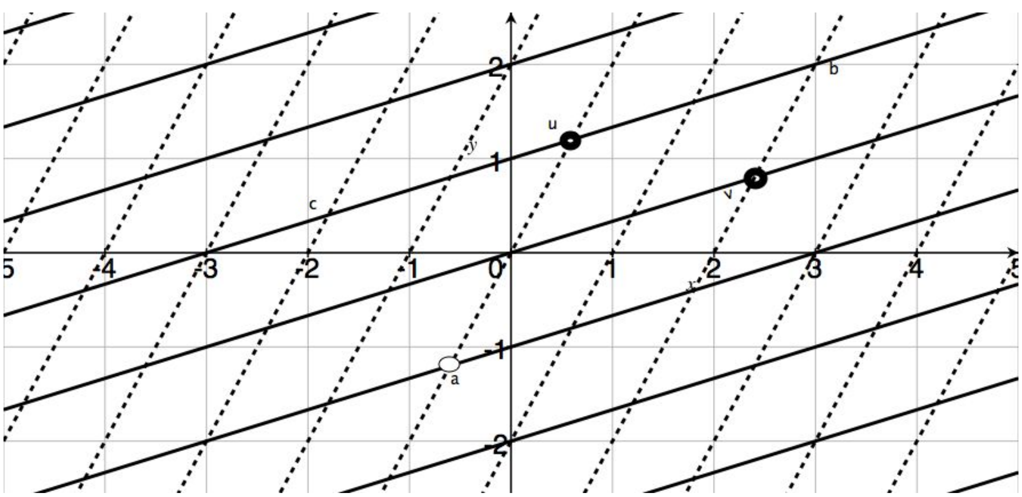
   If the coefficient matrix has a pivot position in every row, then there is a pivot position in the bottom row, and there is no room for a pivot in the augmented column. So, the system is consistent.

7. In a wind tunnel experiment, the force on a projectile due to air resistance was measured at different velocities:

Velocity (100 ft/sec)	0	2	3	6	8	10
Fource (100lb)	0	2.9	14.8	39.6	74.3	119

   Find an interpolating polynomial for the data and estimate the force on the projectile when the projectile is traveling at 750 ft/sec . Use $p(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$. What happens if you try to use a polynomial of degree less than 5?

8. Use the following figure to write $\vec{a}$,$\vec{b}$, and $\vec{c}$ as a linear combination of $\vec{u}$ and $\vec{v}$. Is ever vector in $\mathbb{R}^2$ a linear combination of $\vec{u}$ and $\vec{v}$?



   Label the following vectors in the above figure. $\vec{d} = \vec{v} - 2\vec{u}$, $\vec{e} = 2\vec{u} - 2\vec{v}$.

9. A mining company has two mines. One day's operation at mine #1 produces ore that contains 20 metric tons of copper and 550 kilograms of silver, while one day's Operation at mine #2 produces ore that contains 30 metric tones of copper and 500 kilograms of

silver. Let $\vec{v_1} = [20\ 550]^\top$ and $\vec{v_2} = [30\ 500]^\top$. Then $\vec{v_1}$ and $\vec{v_2}$ represent the "output per day" of mine #1 and mine #2 respectively.

(a) What physical interpretation can be given to the vector $5\vec{v_1}$?

This is the output for 5 days from mine #1

(b) Suppose the company operates mine #1 for x_1 days and mine #2 for x_2 days. Write a vector equation whose solution gives the number of days each mine should operate in order to produce 150 tons of copper and 2825 kilograms of silver.

The total output is $x_1\vec{v_1} + x_2\vec{v_2}$. So x_1 and x_2 should satisfy $x_1\vec{v_1} + x_2\vec{v_2} = [150\ 2825]^\top$

(c) Solve the equation you set up above.

1.5 days for mine #1 and 4 days for mine #2.

10. Construct a 3×3 matrix, not in echelon form, whose columns span $\mathbb{R}^3$. Show that the matrix you constructed has the desires property.

11. Let A be a 3×2 matrix. Explain why the equation $A\vec{x} = \vec{b}$ cannot be consistent for all $\vec{b}$ in $\mathbb{R}^3$. Generalize your argument to the case of an arbitrary A with more rows than columns.

12. Find a parametric equation of the line M through $\vec{p} = [2\ \ -5]^\top$ and parallel to $\vec{q} = [-3\ 1]^\top$. Find the parametric equation of the line L through $\vec{p}$ and $\vec{q}$.

$L = [2\ 5]^\top + t[-5\ 6]^\top$

13. Does the equation $A\vec{x} = \vec{0}$ have a nontrivial solution if

(a) A is a 3×3 matrix with three pivot columns.

When A is a 3×3 matrix with three pivot columns, the equation $A\vec{x} = \vec{0}$ has no free variables and so has no nontrivial solutions.

(b) A is a 3×2 matrix with two pivot columns.

Each column is a pivot column so the equation $A\vec{x} = \vec{0}$ has no friee variables and so no nontrivial solutions.

14. Construct a 3×3 nonzero matrix A such that the vector $[1\ 1\ 1]^\top$ is a solution of $A\vec{x} = \vec{0}$.

15. Find the matrix A so that the corresponding linear operator $T_A : \mathbb{R}^2 \to \mathbb{R}^2$

(a) rotates all points $\frac{\pi}{6}$ radians counterclockwise about the origin.

(b) first reflects all points about the x-axis, then rotates all points $\frac{\pi}{4}$ radians *clockwise* about the origin.

(c) is such that $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}2\\3\end{bmatrix}$ and $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}4\\6\end{bmatrix}$

16. Given an example of two distinct, nonzero vectors in $\mathbb{R}^3$ that are linearly dependent.