

NAME:

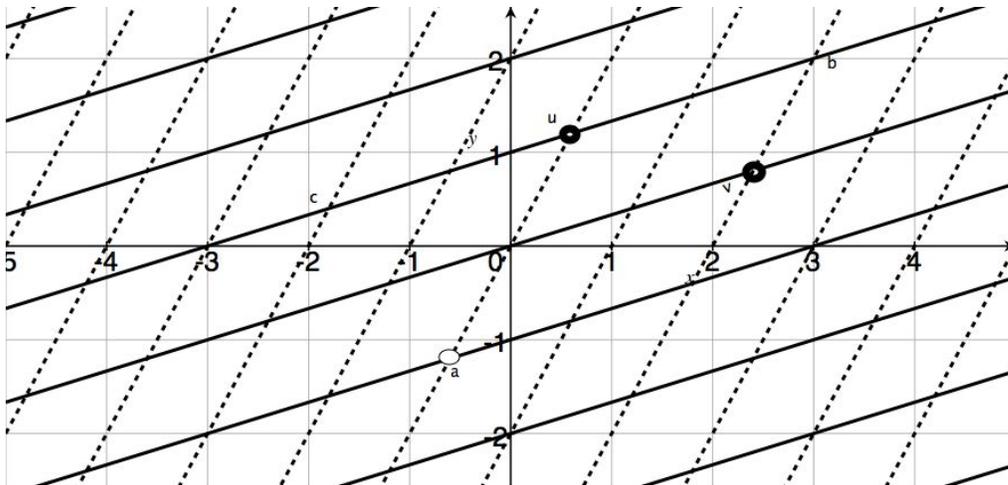
1. Define carefully the following terms:
 - (a) Linearly Independent
 - (b) Spanning Set
 - (c) Basis
 - (d) Vector Space
 - (e) Subspace
 - (f) Linear Transformation/Function/Operator
2. Complete the following for each of the given matrices A .
(Make one up for right now.)
 - (a) §1.4 Find the reduced row echelon form for A .
 - (b) §3.3 Are the columns of A linearly independent?
 - (c) §3.2 Do the columns of A span \mathbb{R}^m where m is the number of rows in A ?
 - (d) §3.4 Find the domain and codomain for the linear transformation T_A .
 - (e) §3.4 Is the linear transformation T_A one-to-one?
 - (f) §3.4 Is the linear transformation T_A onto?
 - (g) §3.3 Find a basis for the column space of A
 - (h) §3.3 Find a basis for the null space of A .
 - (i) §1.3 What is the rank of A ?
 - (j) §3.3 Find a basis for the span of T_A .
 - (k) §3.3 Find a basis for the kernel of T_A .
 - (l) §2.6 If possible, compute the determinant of A .
 - (m) §2.5 If possible, determine if A is invertible, and then find A^{-1} .
 - (n) §5.1 Is $\vec{0}$ and eigenvector of A ?
 - (o) §5.1 If possible, find the characteristic polynomial of A ?
 - (p) §5.1 If possible, find the eigenvalues of A ?
3. §1.1 Do the three lines $x_1 - 4x_2 = 1$, $2x_1 - x_2 = -3$, and $-x_1 - 3x_2 = 4$ have at least one common point of intersection? Explain.

4. §1.4 Suppose a 3×5 coefficient matrix for a system has three pivot columns. Is the system consistent? Why or why not?
5. §1.4 Suppose a system of linear equations has a 3×5 augmented matrix whose fifth column is a pivot column. Is the system consistent? Why or why not?
6. §1.4 Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.
7. §1.1 In a wind tunnel experiment, the force on a projectile due to air resistance was measured at different velocities:

Velocity (100 ft/sec)	0	2	3	6	8	10
Force (100lb)	0	2.9	14.8	39.6	74.3	119

Find an interpolating polynomial for the data and estimate the force on the projectile when the projectile is traveling at 750 ft/sec . Use $p(t) = a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5$. What happens if you try to use a polynomial of degree less than 5?

8. §2.1 Use the following figure to write \vec{a}, \vec{b} , and \vec{c} as a linear combination of \vec{u} and \vec{v} . Is every vector in \mathbb{R}^2 a linear combination of \vec{u} and \vec{v} ?



Label the following vectors in the above figure. $\vec{d} = \vec{v} - 2\vec{u}$, $\vec{e} = 2\vec{u} - 2\vec{v}$.

9. §1.3 A mining company has two mines. One day's operation at mine #1 produces ore that contains 20 metric tons of copper and 550 kilograms of silver, while one day's Operation at mine #2 produces ore that contains 30 metric tons of copper and 500 kilograms of silver. Let $\vec{v}_1 = [20 \ 550]^\top$ and $\vec{v}_2 = [30 \ 500]^\top$. Then \vec{v}_1 and \vec{v}_2 represent the "output per day" of mine #1 and mine #2 respectively.

- (a) What physical interpretation can be given to the vector $5\vec{v}_1$?
- (b) Suppose the company operates mine #1 for x_1 days and mine #2 for x_2 days. Write a vector equation whose solution gives the number of days each mine should operate in order to produce 150 tons of copper and 2825 kilograms of silver.
- (c) Solve the equation you set up above.

10. §3.2 Construct a 3×3 matrix, not in echelon form, whose columns span \mathbb{R}^3 . Show that the matrix you constructed has the desired property.
11. §1.4 Let A be a 3×2 matrix. Explain why the equation $A\vec{x} = \vec{b}$ cannot be consistent for all \vec{b} in \mathbb{R}^3 . Generalize your argument to the case of an arbitrary A with more rows than columns.
12. Find a parametric equation of the line M through $\vec{p} = [2 \ -5]^\top$ and parallel to $\vec{q} = [-3 \ 1]^\top$. Find the parametric equation of the line L through \vec{p} and \vec{q} .
13. §1.4 Does the equation $A\vec{x} = \vec{0}$ have a nontrivial solution if
 - (a) A is a 3×3 matrix with three pivot columns.
 - (b) A is a 3×2 matrix with two pivot columns.
14. §2.2 Construct a 3×3 nonzero matrix A such that the vector $[1 \ 1 \ 1]^\top$ is a solution of $A\vec{x} = \vec{0}$.
15. §2.3 Find the matrix A so that the corresponding linear operator $T_A : \mathbb{R}^2 \rightarrow \mathbb{R}^2$
 - (a) rotates all points $\frac{\pi}{6}$ radians counterclockwise about the origin.
 - (b) first reflects all points about the x -axis, then rotates all points $\frac{\pi}{4}$ radians *clockwise* about the origin.
 - (c) is such that $T \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $T \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$
16. §3.3 Given an example of two distinct, nonzero vectors in \mathbb{R}^3 that are linearly dependent.