Quiz 3

Key

Show *all* your work. No credit is given without reasonable supporting work. There are *two* sides to this quiz.

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, circle F and provide a counterexample or brief reasoning.

nswer (.5)
ect reason (.5)
ct counterex (1)

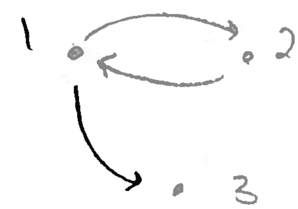
T F (Suggested §4.1 #37b) Let a, b, and c be integers. this should have been a6 (to make it easier)
If $a \equiv b($ mod 6$)$ and $c \equiv d($ mod 6$)$, then $a^c \equiv b^d($ mod 12$)$.

note $2 \equiv 8 \bmod 6$ and $6 \equiv 0 \bmod 6$

but $2^6 = 64 \bmod 12$ and $8^0 \equiv 1 \bmod 12$
$\equiv 4 \bmod 12$ and $1 \not\equiv 4 \bmod 6$

answer (.5)
stat reason (.5)
just counterex (1)

T F (HW5 §9.1 #3) If R is a relation that is not symmetric, then R must be anti-symmetric.



Let R be a relation defined on the set $A = \{1, 2, 3\}$ by the diagram to the right.
Note, R is not symmetric (b/c $3 \not R 1$) but not antisymmetric (b/c $1R2$ and $2R1$ but $1 \neq 2$)

2. Let R be a relation on the integers, $\mathbb{Z}$, defined by aRb if $a \equiv b($ mod 8$)$.

(a) [2] (HW5 §4.1 #4) Identify a number in the congruence class of 3 that is between -25 and -15.

what cong class (1)
in domain requested (.5)
got it (.5)

$3 - 8 = -5$ | $-5 - 8 = -13$ | $-13 - 8 = -21$

$\Rightarrow 3 \equiv -5 ($mod 8$)$ | $\Rightarrow 3 \equiv -13 ($mod 8$)$ | $\Rightarrow 3 \equiv -21 \bmod 8$

So -21 works

(b) [4] (HW5 §9.5 #2) Determine if R is an equivalence relation. If not, explain what property R failed. If so, *briefly* justify that R satisfies the necessary properties.

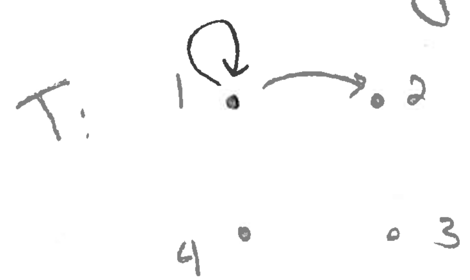
1.5 def of equal
(1.5)
reasoning (1)
sense/notation (1)

Is an equivalence relation b/c R is
reflexive: $\forall x \in \mathbb{Z}$, xRx b/c $x \equiv x \bmod 8$
symmetric: $\forall x, y \in \mathbb{Z}$, if xRy we know $x \equiv y \bmod 8$
$\Rightarrow x - y$ is divisible by 8
so $y - x$ is divisible by 8
$\Rightarrow y \equiv x \bmod 8$ $\therefore yRx$
transitive: $\forall x, y, z \in \mathbb{Z}$ if $x \equiv y \bmod 8$ & $y \equiv z \bmod 8$ then $x \equiv z \bmod 8$

3. [4] Define a relation T that is anti-symmetric, but not symmetric or reflexive. defined well (+1)

(+1) (+1) (+1)

note: there are many answers



anti-symmetric b/c only elements x & y with xTy and yTx is when $x=1=y$

not symmetric b/c $1T2$ but $2 \not T 1$

not reflexive b/c $2 \not T 2$

4. (Suggested §9.1 #3) Consider the relation S on the set $A = \{1, 2, 3, 4\}$ defined by $S = \{(1, 1), (1, 2), (1, 3), (2, 3), (3, 1)\}$

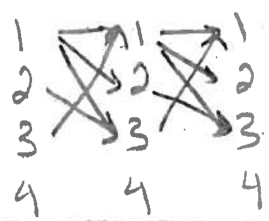
(a) [2] Determine if S is anti-symmetric. Justify your answer.

no (+.5), b/c $\underbrace{1S3 \text{ and } 3S1 \text{ but } 1 \neq 3}_{\text{counter ex (+.5)}}$

def of anti symmetric (+1)

ball park of def +.5

(b) [2] (relations wks #4) Find $S \circ S$ or S^2



sketch/idea (+.5)

$\{(1,1), (1,2), (1,3), (2,1), (3,1), (3,2), (3,3)\}$

set (+1)

miss 1 element +.5

def of $S \circ S$ or S^2 (+.5)

(c) [2] (Suggested §9.4 #3) Find the symmetric closure of S.

note: $1S2$ but $2 \not S 1$ so we'll add $(2,1)$ (+.5)

$2S3$ but $3 \not S 2$ so we'll add $(3,2)$ (+.5)

So the symmetric closure of S is:

$\{(1,1), (1,2), (1,3), (2,3), (3,1), (2,1), (3,2)\}$ or

not any larger (+.5)[2]

includes S (+.5)