Quiz 2

Key

Show *all* your work. No credit is given without reasonable supporting work. There are *two* sides to this quiz.

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, circle F and provide a counterexample or brief reasoning.

answer (+.5)
start (+.5)
reasoning (+1)

T F $\{2, 8\} \in \{-3, 2, 0, 8.3\}$

Notice even $\{2,8\} \nsubseteq \{-3,2,0,8.3\}$

But it is also not true that the ~~set~~ set $\{2,8\}$ is contained/an element of $\{-3,2,0,8.3\}$

answer (+.5)
start (+.5)
reasoning (+1)

T F $24.3 \in \{x \in \mathbb{R} \,|\, 1 \leq 2x < 58\}$

Note $2 \cdot 24.3 = 48.6$

and $1 \leq 48.6 < 58$

So $24.3 \in \{x \in \mathbb{R} \mid 1 \leq 2x < 58\}$

start (+.5)
intro/plan (+.5)

2. [6] (Suggested §2.1 #25) Let A and B be sets.
Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ if and only if $A \subseteq B$.

We will prove both directions of the if and only if:

$A \subseteq B \Rightarrow P(A) \subseteq P(B)$:

Let $X \in P(A)$ we want to show $X \in P(B)$.

Since $X \in P(A)$ we know X is a subset of A ie $X \subseteq A$.

Notice $X \subseteq A \subseteq B \Rightarrow X \subseteq B$ (by transitivity).

Thus X is a subset of B & an element of $P(B)$ by definition of powerset.

(+2)
this direction +1
clear +1

define trans +.5

Know P(A) +.5

$P(A) \subseteq P(B) \Rightarrow A \subseteq B$:

We will prove the contrapositive: $A \nsubseteq B \Rightarrow P(A) \nsubseteq P($

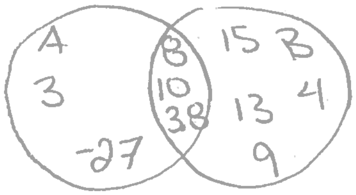
Since $A \nsubseteq B$, there exists a $y \in A$ that is not an element of B.

Consider $\{y\} \subseteq A$. Thus $\{y\} \in P(A)$ but b/c $y \notin B$, $\{y\} \notin P(B)$. Thus $P(A) \nsubseteq P(B)$

(+2)
this direction +1
clear +1

[illegible]

3. [3] (HW3 §2.2 #1) If A and B are sets such that $A \backslash B = \{3, -27\}$, $B \backslash A = \{15, 4, 13, 9\}$, and $A \cap B = \{8, 10, 3.8\}$, find the sets A and B.



$A = \{-27, 3, 3.8, 8, 10\}$

$B = \{3.8, 4, 8, 9, 10, 13, 15\}$

4. (Suggested §2.2 #29) What can you say about the sets A and B if we know that:

(a) [2] $A \cup B = A$? Justify your answer.

(+1) $B \subseteq A$.

(+1) The union by definition is $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$. If $A \cup B = A$, then all elements from B must have already been added from considering the set A.

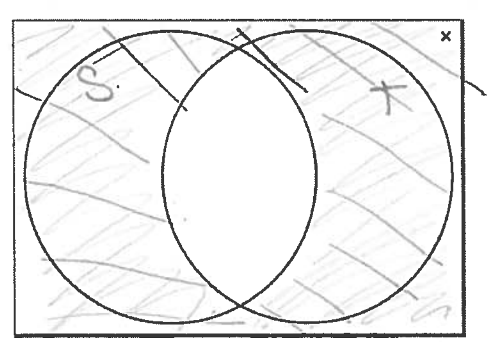
(b) [3] $A \setminus B = B \setminus A$? Justify your answer.

(+1) note $A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$ def (+1)
$B \setminus A = \{y \mid y \notin A \text{ and } x \in B\}$.
If $x \in A \setminus B$ and $x \in B \setminus A$, that means $x \in A$ and $x \notin A$ ~~A~~
thus $A \setminus B = \varnothing$ (+1) $\Rightarrow A = B$.

5. (set wks #7) Let the venn diagram be of the sets S and T below in the universal set U. The set S is the one on the left and the set T is the one on the right.

(a) [1] Identify (by shading) the set $\overline{S \cap T}$.

(b) [1] Use Set Identities (such as distributive or deMorgan's laws) to write the set in part (a) in a different manner.



By deMorgan's law:

$\overline{S} \cup \overline{T}$