

1. [3] Let  $a$ ,  $b$ , and  $c$  be integers, where  $a \neq 0$ . Prove that if  $a|b$  and  $b|c$ , then  $a|c$ .
2. [3] Let  $a$ ,  $b$ , and  $c$  be positive integers and  $a \neq 0$ . Prove or disprove if  $a|(bc)$  then  $a|B$  or  $a|c$ .
3. [2] Let  $a$  and  $b$  be integers such that  $a \equiv 11 \pmod{19}$  and  $b \equiv 3 \pmod{19}$ . Show work done by hand or Sage code used to find:
  - (a) the integer  $c \in [0, 18)$  such that  $c \equiv 13a \pmod{19}$
  - (b) the integer  $d \in [0, 18)$  such that  $8d \equiv 8b \pmod{19}$
4. [2] Show work done by hand or Sage code used to find the integer  $a$  such that:
  - (a)  $a \equiv 43 \pmod{23}$  and  $-22 \leq a \leq 0$
  - (b)  $a \equiv -1 \pmod{23}$  and  $90 \leq a \leq 110$ .

1. [3] Recall that congruence modulo 16 is an equivalence relation.
  - (a) Identify five numbers in the equivalence class of  $-2 \pmod{16}$ .
  - (b) Identify five numbers in the equivalence class of  $3 \pmod{16}$ .
  - (c) Use set builder notation to describe all the elements in the equivalence class of  $3 \pmod{16}$ .
2. [4] Let  $R$  be the relation of logical equivalences on the set of all compound propositions.
  - (a) Show  $R$  is an equivalence relation.
  - (b) Identify two elements that are in the equivalence class of  $p \rightarrow q$ .
3. [3] Define an equivalence relation on the set of classes offered at UWT.
  - (a) Justify why your relation is an equivalence relation.
  - (b) Determine the equivalence classes for your relation.

1. [3] Determine whether the relation  $R$  on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where  $(x, y) \in R$  if and only if:
  - (a)  $x + y = 0$
  - (b)  $x = 2y$
  - (c)  $x = 1$  or  $y = 1$
2. [5] Let  $R_5$  and  $R_6$  be the “congruent modulo 5” and the “congruent modulo 6” relations, respectively, on the set of integers. That is  $R_5 = \{(a, b) | a \equiv b \pmod{5}\}$  and  $R_6 = \{(a, b) | a \equiv b \pmod{6}\}$ . Find:
  - (a)  $R_5 \cup R_6$
  - (b)  $R_6 \setminus R_5$
  - (c)  $R_5 \oplus R_6$
  - (d)  $R_5 \circ R_5$
  - (e)  $R_5 \circ R_6$
3. [2] Define a relation on the set  $A = \{0, 1, 2, 5\}$  that is neither symmetric nor antisymmetric.

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2. [3] Find the symmetric closures of the relations with the directed graph shown in problem 1.
3. [4] Let  $R$  be the relation on the set  $\{1, 2, 3, 4, 5\}$  containing the ordered pairs  $(1, 3)$ ,  $(2, 4)$ ,  $(3, 1)$ ,  $(3, 5)$ ,  $(4, 3)$ ,  $(5, 1)$ ,  $(5, 2)$ , and  $(5, 4)$ . Find the transitive closure of  $R$ .