§2.5 # WrittenHW #4 TCSS 321

1. [3] Determine whether each of these sets is finite, countably infinite, or uncountable. Justify your answers.
   (a) integers greater than 100
   (b) the set $A \times \mathbb{Z}$, where $A = \{-4, \text{Apple}\}$.
   (c) the real numbers between 0 and 2
2. [2] Give an example of two uncountable sets $A$ and $B$ such that $A \setminus B$ is countably infinite.
3. [2] Give an example of two uncountable sets $A$ and $B$ such that $A \cap B$ is finite.
4. [3] Suppose that Hilbert's Grand Hotel is fully booked. The hotel, however needs to close all the odd numbered rooms for maintenance. Can the hotel accommodate all its current guests and still complete the maintenance? If possible, describe the algorithm.

§3.1 # TCSS 321 Winter 2013

1. [3] Describe and algorithm that takes as input a list of $n$ distinct integers and finds the location of the largest even integer in the list or returns 0 if there is no even integers in the list.
2. [2] Use sage to code the above algorithm and run the algorithm on the lists below. Attach the code and the outputs when given the below lists to this assignment.
   (a) $\{-2, 5, -10, 23, -12\}$
   (b) $\{-13, 3, 5, -71, 91, 33\}$
3. [5] A palindrome is a string that reads the same forwards as it does backwards. Create and describe and algorithm for determining whether a string of $n$ characters is a palindrome.

§2.3 # TCSS 321 Winter 2013

1. [2] Determine whether $f$ is a function from $\mathbb{Z}$ to $\mathbb{R}$ and justify your answer.
   (a) $f(n) = \sqrt{n^2+1}$
   (b) $f(n) = \dfrac{1}{n^2-9}$
2. [2] Create a function that is not the identity function, from $\mathbb{Z}$ to $\mathbb{Z}$ that has an inverse.
3. Let $f(x) = \lfloor x \rfloor + \lceil x/3 \rceil$ be a map from $\mathbb{R}$ to $\mathbb{R}$.
   (a) [1] Evaluate $f(5)$.
   (b) [1] Evaluate $f(-13)$.
   (c) [2] Identify the image/range of $f$.
   (d) [2] Graph $f$.

§3.2 # TCSS 321 Winter 2013

1. [2] Show that $x^3$ is $O(x^5)$ but $x^5$ is not $O(x^3)$.
2. [3] Arrange the functions , $\sqrt{n}$, $1000 \log n$, $n \log n$, $2n!$, $2^n$, $3^n$, and $\frac{n^2}{10000}$ in a list so that each function is bit-$O$ of the next function.
3. [3] Show that $1^4 + 2^4 + 3^4 + 4^4 + ... + n^4$ is $O(n^5)$.
4. [2] Suppose that you have two different algorithms for solving a problem. To solve problem of size $n$, the first algorithms uses exactly $n^2 2^n$ operations and the second algorithm uses exactly $n!$ operations. AS $n$ grows, with algorithm uses fewer operations? Justify yourself.