§1.5 # WrittenHW #2 TCSS 321

1. [1] Let $C(x, y)$ stand for "person x is enrolled in class y", where the domain for x is all students in your school and the domain for y is all classes offered at your school. Express the statement: $\exists x(C(x, TMath125) \land C(x, TCCS305))$ in a simple English sentence.

2. [2] Let $F(x, y)$ mean that x can fool y, where the domain consists of all people in the world. Use quantifiers to express each of these statements.

   (a) Everybody can fool Sammy.

   (b) Everyone can be fooled by somebody.

3. [2] Use predicates, quantifiers, logical connectives, and mathematical operators to express the statement that there is a positive integer that is not the sum of three squares.

4. [2] Determine the truth value of each of these statements if the domain of each variable consists of all really numbers.

   (a) $\forall x \exists y(x^2 = y)$

   (b) $\forall x \exists y(x = y^2)$

5. [3] Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

   (a) $\neg \exists y \exists x P(x, y)$

   (b) $\neg \exists y(Q(y) \land \forall x \neg R(x, y))$

   (c) $\neg \exists y(\forall x \exists z T(x, y, z) \lor \exists x \forall z U(x, y, z))$

1. [3] Use the formal rules of inference to show that the hypotheses:

   (a) "If it does not snow or if it is not foggy, then the race will be held and the lifesaving demo will go on."
   (b) "If the race is held, then the trophy will be awarded."
   (c) "The trophy was not awarded."

   to conclude that "It snowed."

2. [2] For the following set of premises, what relevant conclusion or conclusions can be drawn? Explain yourself using the formal rules of inference.

   (a) "All foods that are healthy to eat do not taste good."
   (b) "Tofu is healthy to eat."
   (c) "You only eat what tastes good."
   (d) "Cheeseburgers are not healthy to eat."

3. [2] For each of the arguments below, determine whether the arguments is correct or incorrect. Explain why.

   (a) All students in Discrete 1 understand logic. Peter is a student in this class. Therefore, Peter understands logic.
   (b) Every computer science major takes Discrete 1. Xavier is taking Discrete 1. Therefore, Xavier is a computer science major.

4. [3] Identify the error or errors in this argument that supposedly shows that:
   if $\forall x(P(x) \vee R(x))$ is true, then $\forall x P(x) \vee \forall x R(x)$ is true.

1. $\forall x(P(x) \vee R(x)$	Premise
2. $P(c) \vee R(c)$	Universal instantiation form (1)
3. $P(c)$	Simplification from (2)
4. $\forall x P(x)$	Universal generalization from (3)
5. $R(x)$	Simplification from (2)
6. $\forall x R(x)$	Universal generalization from (5)
7. $\forall(P(x) \vee \forall x R(x))$	Conjunction from (4) and (6)

1. [2] Consider the following steps for finding the solutions to $\sqrt{x+3} = x - 3$. Determine if the steps are correct, or not. If not, identify what step(s) contain the error.

   (a) $\sqrt{x+3} = x - 3$ is given;
   (b) $x + 3 = x^2 - 6x + 9$ by squaring both sides of 1.
   (c) $0 = x^2 - 7x + 6$ obtained by subtracting $x + 3$ from both sides of 2.
   (d) $0 = (x - 1)(x - 6)$ obtained by factoring the right-hand side of 3.
   (e) $x = 1$ or $x = 6$ with follows from 4, because $ab = 0$ implies that $a = 0$ or $b = 0$.

2. [3] Prove that if n is a positive integer, then n is odd if and only if $5n + 6$ is odd.

3. [5] Show if you pick three socks from a bin containing just red and white socks, you must get either a pair of red socks or a pair of white socks.

1. [3] Prove or disprove that there is a positive integer that equals the sum of the positive integers not exceeding it.

2. [5] Prove or disprove that if x and y are real numbers then $|x| + |y| \geq |x + y|$.

3. [2] Prove or disprove that you can use dominoes to tile a standard checkerboard with the four corners removed.