TCSS 321

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, circle F and provide a counterexample or brief reasoning.

   T F If A and B are sets then $A \setminus B = A \cap \overline{B}$

   T F If $a|(bc)$ where a, b, c are integers greater than zero, then $a|b$ or $a|c$.

   T F A relation on a set cannot be both symmetric and antisymmetric.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit. We use the logical symbols from the textbook unless otherwise specified.

2. (Quiz1 #3) Consider the following statement:
   "There is a girl who eats curds and whey."

   (a) [4] Express the statement above using predicates, quantifiers, and logical connectives. Be sure to *define* any terms your create.

   (b) [2] Negate part (a) so that no negations are to the left of any quantifiers.
   Make use of DeMorgan's law.

3. (proof con't wks #1) Read the following "Theorem". Determine (and justify)

   (a) [2] if the "Theorem" is true

   (b) [4] if the "Proof" is valid.

Theorem 1 *Let m, n, and p be integers. If $m+n$ and $n+p$ are even integers, then $m+p$ is an even integer.*

Proof 1 *Assume that $m+n$ and $n+p$ are even integers. We want to show that $m+p$ is an even integer.*

Since $m+n$ is an even integer, there exists integers a and b such that

$$m+n = 2a+2b.$$

Thus we know that $m = 2a$ and $n = 2b$.

Since $n+p$ is an even integer and $n = 2b$, we know that there exists and integer c such that

$$n+p = 2b+2c.$$

Thus we also know that $p = 2c$.

Now we can consider $m+p$ which equals $2a+2c = 2(a+c)$. Thus $m+p$ is even.

4. (§2.4 #17) Consider the sequence $\{a_n\}$ where $a_n = 2 \cdot n \cdot a_{n-1}$ where $a_0 = 1$.

   (a) [3] Write the first five terms of the sequence.

   (b) [4] Find a closed form (non-recursive definition) for a_n when $n \geq 1$.

5. Consider the recursive algorithm described below for the next few questions.

```
int RecursiveAlg(n)
Data: n: integer where n > 0
if n == 1 then
    return 1;
else
    return n· RecursvieAlg(n-1)
end
```

Algorithm 1: Recursive Algorithm

(a) [2] (Exam2 #4) Describe what the above algorithm does.

(b) [3] (recursive wks #5) Use induction to prove the recursively defined program computes what you claimed in part (a).

(c) [4] (§3.1 #3) Write a program that is *not* recursive but completes the same computations done above.

(d) [2] (§3.3 #) Give a big-Θ estimate for the number of products taken in your algorithm from part (c). Show your reasoning.

6. [4] (Exam2 #7) Find $112_3 \times 210_3$. Give your answer in decimal/base 10.

7. [4] (§4.3 #31) The product of two integers is $2^4 \cdot 3^5 \cdot 7^4 \cdot 11$ and their greatest common divisor is $2^4 \cdot 3^2 \cdot 11$. What is the least common multiple of the two integers? Show your reasoning.

8. [5] (11/14 Lecture) Solve the system of congruences $x \equiv 6 \mod 8$ and $x \equiv 3 \mod 7$

9. [5] Choose *ONE* of the following and support your conclusions. Clearly identify which of the two you are answering and what work you want to be considered for credit.

   (a) (§5.1 #32) Prove 3 divides $n^3 + 2n$ whenever n is a positive integer.

   (b) (§5.2 #3) Prove that a postage of n cents, for $n \geq 8$ can be formed using just 3-cent stamps and 5-cents stamps.

10. [4] (functions wks #5) Define a function that is onto/surjective, but not one-to-one.

11. [6] Choose *ONE* of the following and support your conclusions. Clearly identify which of the two you are answering and what work you want to be considered for credit.

    (a) (11/14 lecture) Show if a, b, and m are integers such that $m \geq 2$ and $a \equiv b \mod m$, then the gcd(b, m) divides the gcd$(a.m)$.

    (b) (§9.1 #50) Let R and S be reflexive relations on a set A. Show that $R \oplus S$ is irreflexive.

12. [4] (12/5 lecture) Define a set and a relation on that set that is a partial ordering but not a total ordering.

13. [2] My favorite topic/activity in the course was:

14. [6] Choose *ONE* of the following and support your conclusions. Clearly identify which of the two you are answering and what work you want to be considered for credit.

    (a) (Exam1 #8) Suppose you are on an island that has two kinds of inhabitants, knights who always tell the truth, and their opposites, knaves who always lie. You encounter two people A and B. What are A and B if A says "I am a knave or B is a knight." and B says nothing.

    (b) (Exam2 #6) Clearly describe an algorithm that finds all terms of a finite sequence of integers that are greater than the sum of all previous terms in the sequence.