# TCSS 321



1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, circle F and provide a counterexample or brief reasoning.

   T F $\{2, -4\} \in \{8.3, -4, 7.2, 4, 2, 0\}$.

   T F Let $A$ and $B$ be sets, then $A \setminus B = A \cap \overline{B}$.

   T F Let $A$, $B$, $C$, and $D$ be sets, then $(A \times B) \times (C \times D) = A \times (B \times C) \times D$.

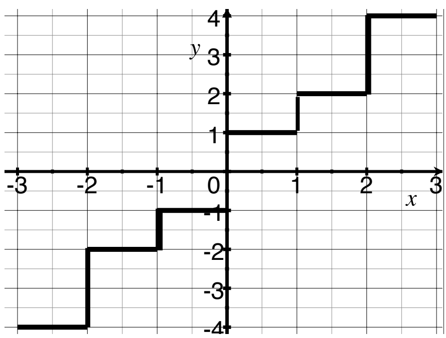
Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

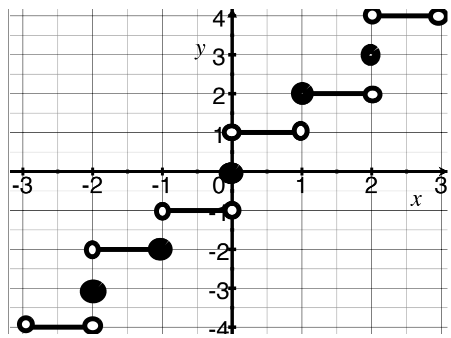
2. [3] (HW3 §2.5 #3) Give an example of two uncountable sets $A$ and $B$, such that $A \cap B$ is finite.

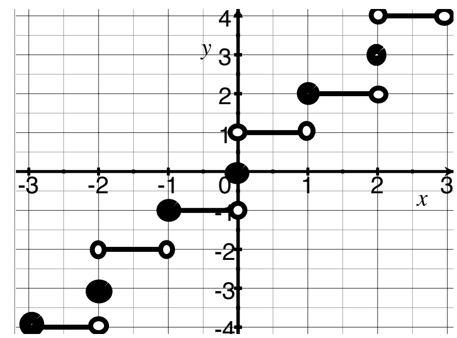
3. (HW4 §2.3 #3) Let $f(x) = \lfloor x \rfloor + \lceil x/2 \rceil$ be a map from $\mathbb{R}$ to $\mathbb{R}$.

   (a) [2] Evaluate $f(-2)$.

   (b) [2] Which of the following is a graph of $f$? Briefly justify yourself.








4. Each of the following give an exact count of operations taken in a worse case senerio for a certain algorithm. Identify their complexity (big-O for programmers or big-$\Theta$ for computer scientists) as you would to a colleague and justify yourself.

   (a) [2] (§3.2 #5) $\dfrac{x^2+1}{x+1}$

   (b) [2] (HW4 §3.2 #2) $\sqrt{n} + \log n$

   (c) [2] (HW4 §3.2 #3) $1 + 2 + 3 + ... + n$

5. (§2.4 #17) Consider the sequence $\{a\}$ defined recursively by $a_n = a_{n-1} + 2$ where $a_0 = 3$.

   (a) [2] Find $a_1$ and $a_2$.

   (b) [4] Find a (closed form) formula for $a_n$ as a function of $n$ (as opposed to using any terms previous to $a_n$ in the formula).

6. [3] (§2.4 #33) Compute $\displaystyle\sum_{i=0}^{2}\left(\sum_{j=0}^{3}(2i+3j)\right)$.

7. [6] (matrix wks) Let $M = \begin{bmatrix} a & c \\ 0 & c \\ 0 & 0 \end{bmatrix}$, $N = \begin{bmatrix} 1 & 0 & b \\ 0 & f & a \end{bmatrix}$, and $P = \begin{bmatrix} 1 & 0 \\ 0 & f \\ f & 0 \end{bmatrix}$,
   where $a$, $b$, $c$, and $f$ are nonzero real numbers. Find the following if possible:

   $M + P$ $\qquad$ $M^T + P$ $\qquad$ $NP$

8. (matrix wks #6) Consider the algorithm described in pseudocode below for the next three questions.

**Data**: $A$, $B$: $n \times n$ matrices where $n > 2$
**for** $i = 1$ *to* $n$ **do**
  **for** $j = 1$ *to* $n$ **do**
    $c_{ij} := 0$
    **if** $i \le j$ **then**
      **for** $q := i$ *to* $j$ **do**
        $c_{ij} := c_{ij} + a_{iq}b_{qj}$
      **end**
    **end**
  **end**
**end**
return $C$ #$C = [c_{ij}]$

**Algorithm 1**: Matrix Algorithm

(a) [4] (§3.1 #1) Use Algorithm 1 by hand on $A = \begin{bmatrix} a & b \\ 0 & b \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$. Clearly indicate your computations for each $i$ and $j$.

(b) [2] Describe what the algorithm is doing as you would to a colleague during lunch.

(c) [4] (§3.3 #3) Give a big-O estimate for the number of multiplications used in Algorithm 1 as a function of $n$. Show your reasoning.

9. [6] Choose *ONE* of the following and support your conclusions. Clearly identify which of the two you are answering and what work you want to be considered for credit. Let $A$ and $B$ be sets.

   (a) (Quiz2 #2) Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ if and only if $A \subseteq B$.

   (b) (HW3 #3) Prove if $\mathcal{P}(A) = \mathcal{P}(B)$, then $A = B$.