11:00
10:34
26 min

EXAM 1 — TCSS 321 — Winter 2013

1. [12] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, circle F and provide a counterexample or brief reasoning. The logic symbols make use of the textbook notation.

(T) F Let n and m be integers. $\forall n \exists m (n^2 < m)$.

True Let $m = n^2 + 1$

true (+.5)
start reasoning (+.5)
justify/logic (+1.5)
sense (+.5)

T (F) Let $H(x)$ be "x is happy". Given the premise $\exists x H(x)$, we conclude that $H(\text{Lola})$. Thus, Lola is happy.

We only know there exists someone who is happy, that person may not be Lola (she may not even be in the domain!)

false (+.5)
start looking for reason (+.5)
sense (+.5)
counterex/justify (+1.5)

(T) F If n is and odd integer, then n^2 is odd.

Since n is odd $\exists k \in \mathbb{Z} \ni n = 2k+1$

Note $n^2 = (2k+1)^2 = (2k+1)(2k+1) = 4k^2 + 2k + 2k + 1$

$= 2(2k^2 + k + k) + 1$ which is odd

true (+.5)
start reasoning (+.5)
sense (+.5)
justify/logic (+1.5)

(T) F There is a positive integer that equals the sum of the positive integers not exceeding it.

Consider 1

the positive integers not exceeding it are: $\{1\}$

and $1 = 1$ ✓

true (.5)
start reasoning (+.5)
sense (+.5)
justify/logic (+1.5)

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Let the domain be webpages posted on the internet.

   (a) [2] (Quiz1 #1) Write a proposition.

   It will not rain today.

   Start (+.5) sentence (+.5) T/F (+1)

   (b) [3] (HW1§1.4 #4) Write a propositional function P of x where x is in your domain.

   Let P(x) be "x functions correctly".

   Start (+.5) sentence (+.5) T/F (+1) function of x (+1)

3. [5] (HW1 §1.3 #3) Find a compound proposition involving the propositional variables a, b, and c that is false when exactly two of a, b, and c are true, and true otherwise.

understand (+1)

True when: a=T, b=T, c=T; a=T, b=F, c=F; a=F, b=T, c=F; a=F, b=F, c=T; a=F, b=F, c=F

write one situation (+1.5)

4 other situations (+2) $(a\wedge b\wedge c)\vee(a\wedge\neg b\wedge\neg c)\vee(\neg a\wedge b\wedge\neg c)\vee(\neg a\wedge\neg b\wedge c)\vee(\neg a\wedge\neg b\wedge\neg c)$

connection w/ or (+.5)

or

negation of Quiz1 #2 $\neg[(p\wedge q\wedge\neg r)\vee(p\wedge\neg q\wedge r)\vee(\neg p\wedge q\wedge r)]$

4. If you were promoted, then you bought the boss lunch.

   (a) [2] (HW1 §1.1 #3) Write a different, but logically equivalent (English) statement as that given above.

   (+.5) start

   (+.5) equivalent

   It is necessary to buy the boss lunch to be promoted.

   You bought the boss lunch, if you were promoted.

   If you did not buy the boss lunch, then you were not promoted. ← contrapositive

   Many correct answers

   (b) [2] (Quiz1 #1b) Given the above and that you bought the boss lunch, what conclusions (if any) can be drawn? Justify yourself.

   (+.5) No conclusions can be drawn.

   start justify (+.5)

   justify (+1)

   Symbolically if p is "you were promoted" and q is "you bought the boss lunch"

p	q	p→q
F	F	T
F	T	T
T	F	F
T	T	T

We were given p→q and q.

We are in one of the boxed situations in the truth table

5. Let $L(x, y)$ be the statement "x loves y".

   (a) [3] (Suggested §1.5 #9) Express the statement 'Nobody loves everybody'" using quantifiers, and logical connectives. Domain: all people

   There is no one who loves everybody
   There exists no x such that x loves anybody } working towards (+.5)
   $\neg[\exists x \ \forall y \ L(x,y)]$ or $\forall x \ \exists y \ \neg L(x,y)$
   (+.5) (+.5) (+.5) or (+1) (+.5) (+.5) (+.5) or (+1)

   (b) [2] (Suggested §1.5 #31) Negate part (a) so that any negation symbols immediately precede predicates.

   negate (+.5) got it (+.5)

   $\neg(\neg[\exists x \ \forall y \ L(x,y)])$ or $\neg(\forall x \ \exists y \ \neg L(x,y))$
   $\exists x \ \forall y \ L(x,y)$ $\exists x \ \forall y \ \neg(\neg L(x,y))$
   (+.5) (+.5)

6. (§1.7 Ex18) For the following "Theorem", determine:

   (a) [2] if the "Proof" is valid, and

   (b) [4] if the "Theorem" is true. If the "Theorem" is false, provide a counter example, and if the "Theorem" is true but the proof is not valid, provide a proof.

Theorem 1. *If n^2 is an even integer then n is even.* an integer.

Proof. Since n^2 is even there exists an integer a so that $n^2 = 2a$. Let $n = 2b$ for some integer b. This shows that n is even. ☐

(a) { Not valid (+1.5)
{ The proof assumed the conclusion in the underlined location. (+.5) justification

(b) { True. (+1)
Note the contrapositive: if n is not even then n^2 is not even
or if n is odd then n^2 is odd

This was proven true in the 3rd T/F question

read proof (+.5)
logic (+1)
sense (+1)
style (+.5)

If think theorem is false: look for counter ex (+1)
start (+.5)

7. [6] Choose *ONE* of the following and support your conclusions. Clearly identify which of the two you are answering and what work you want to be considered for credit.

start (+1)
explination (+1)
logic (+2)
notation (+1)

(a) (HW1 §1.2 #2) You encounter two people, A and B on an island inhabited by only knights and knaves. The knights always tell the truth and knaves always lie. A says "The two of us are both knights" and B says "A is a knave." If possible, determine what these two people are and justify your conclusions.

(b) (argument wks #2) Three logicians walk into a bar. The bar tender asks "Does everyone want a beer?". The first logician says, "I don't know". The second logician says, "I don't know". The third logician says, "Yes." Explain the joke.

a) The four possibilities enumerated are:

A	B	
knight	knight	(1)
knight	knave	(2)
knave	knight	(3)
knave	knave	(4)

(1) is not possible b/c B would be lying but B is a knight
(2) is not possible b/c A would be lying but A is a knight
(3) is possible
(4) is not possible b/c B would be saying the truth even though he is a knave

A is a knave
B is a knight

(b) Initially the 1st logician would like a beer but if he says yes and the other logicians don't want a beer he would be wrong. Thus he doesn't know enough information to answer. Similarly for the 2nd logician.
Note if either the 1st or 2nd didn't want a beer they could have answer right away with a no — because at least one of them didn't want a beer.

8. [7] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

start (+1)
intro/plan (+1)
definitions (+1)
logic (+2)
clarity (+1)
style (+1)

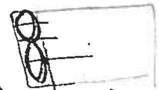
(a) (Suggested §1.8 #43) Prove that you can use dominoes to tile a rectangular checkerboard with an even number of squares.

(b) (EC Presentations #8) Prove either $2 \cdot 10^{400} + 10$ or $2 \cdot 10^{400} + 11$ is not a perfect square.

a) Let the checkerboard have m rows and n columns. Then there are $m \cdot n$ squares on the checkerboard.

We were told $m \cdot n$ is even
Thus either m is even or n is even.

If m is even, we can fit a whole # of dominoes in each column oriented vertically ex.



If n is even we can fit a whole # of dominoes in each row oriented horizontally.

(b) If $2 \cdot 10^{400} + 10$ is not a perfect square then we're done.
If $2 \cdot 10^{400} + 10$ is a perfect square then $\exists\, a \in \mathbb{Z} \ni a^2 = 2 \cdot 10^{400} + 10$
Notice that $(a+1)^2$ is the next possible perfect square but
$(a+1)^2 = a^2 + 2a + 1 = 2 \cdot 10^{400} + 10 + 2a + 1$
$= 2 \cdot 10^{400} + 11 + 2a > 2 \cdot 10^{400} + 11$
Thus $2 \cdot 10^{400} + 11$ cannot be a perfect square.