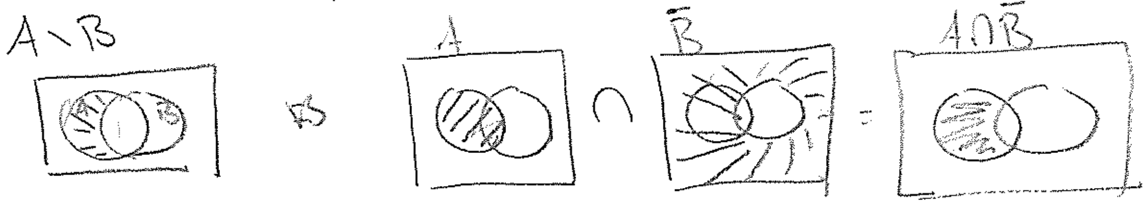
Key

Quiz 2

TRUE/FALSE: Write "True" in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, write "False" and provide a counterexample or brief reasoning. The logic symbols make use of the textbook notation.

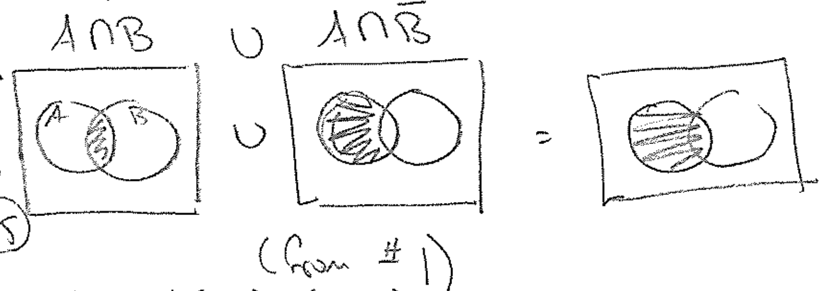
1. [2] (§2.2 #19) Given sets A and B, $A \setminus B = A \cap \overline{B}$

True
.5
logic (1)
sense (.5)



2. [2] (§2.2 #19) Given sets A and B, $(A \cap B) \cup (A \cap \overline{B}) = A$

True
.5
logic (1)
sense (.5)



3. [2] (HW3 §2.1 #2) $\{1, 2\} \in \{1, 2, 3\}$.

False — The set $\{1,2,3\}$ contains no sets

.5
logic (1)
sense (.5)

4. [2] (HW3 §2.1 #2) $\{1, 2\} \subset \{1, 2, 3\}$.

True — $1 \in \{1,2,3\}$ and $2 \in \{1,2,3\}$

so $\{1,2\}$ is a subset of $\{1,2,3\}$

.5
logic (1)
sense (.5)

Free Response: Show your work! No credit is given without supporting work.

Note: there are many correct answers

5. [4] (HW3 §2.5 #3) Find an example of two uncountable sets A and B such that

(a) $A \cap B$ is finite. (+.5) A or B is uncountable

(.5) A+B are well defined

$A = \{x \in \mathbb{R} \mid x \geq 0\}$ note $A \cap B = \{1\}$

intersection (+.5)

is finite (+.5) $B = \{x \in \mathbb{R} \mid x \leq 0\}$

which is finite

(b) $A \cap B$ is countably infinite.

A or B (+.5) uncountable $A = \mathbb{Z} \times \{x \in \mathbb{R} \mid x \geq 0\} = \{(y,x) \mid y \in \mathbb{Z},\ x \in \mathbb{R} \text{ and } x \geq 0\}$

A+B are well def (+1) $B = \mathbb{Z} \times \{x \in \mathbb{R} \mid x \leq 0\}$

is countably inf (+.5) note $A \cap B = \mathbb{Z}$

6. [4] (HW3 §3.1 #3) A palindrome is a string that reads the same forwards as it does backwards. Create and describe an algorithm for detecting if a string can be reordered into a palindrome.

idea (+1)

we'd like to make sure each letter shows up an even number of times, but there might be one letter that shows up an odd number of times that could be put in the middle

sense (+.5)

(+2.5)

```
def: PossiblePalindrome (string)
  odd count = 0
  ABCcount := [0][0]
  for i=0 to len(string)
      usedflag := 0
      for j = 0 to len(ABCcount[0])-1
          if string[i] = ABCcount[j,0]
              (ABCcount[j,1]+1) % 2
              usedflag = 1
      if usedflag = 0
          append string[i] to ABCcount
  for j=0 to len(ABCcount[0])
      oddcount = oddcount + ABCcount[j,1]
  if oddcount > 1
      return "No"
  else return "Yes"
```

7. [4] (§2.6 #37) Justify why the set of all computer programs in a particular programming language is countable.

programs are just letters arranged in words gathered together

letters are countable

There are only a countable # of programs with n letters

Thus all programs would be the countable union of the countable possible programs or

all programs with 1 letter $\cup$ all programs with 2 letters $\cup$ all programs with 3 letters $\cup$ ...

which is countable