EXAM 1 — TCSS 321 — PRACTICE

1. TRUE/~~FALSE~~: Circle T in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, circle F and provide a counterexample or brief reasoning. The logic symbols make use of the textbook notation.

T F $\neg p \vee (r \to \neg q) \equiv \neg p \vee \neg q \vee r$ FALSE

partial truth table

p	r	q	$\neg p \vee (r \to \neg q)$	$\neg p \vee \neg q \vee r$
T	F	F	T	T
T	F	T	T	F
T	T	F	T	T
T	T	T	F	T

differ in two scenarios! NOT logically equivalent.

T F Let n and m be integers. $\forall n \exists m (n^2 < m)$.

True Let $m = n^2+1$

Then for all n it is true that $n^2 < m = n^2+1$

T F Let $H(x)$ be "x is happy". Given the premise $\exists x H(x)$, we conclude that $H(\text{Lola})$. Thus, Lola is happy.

FALSE We know there exists someone who is happy, that person may not be Lola!

(Lola may not even be in the domain!)

T F If n is an even integer, than $n+4$ is even.

TRUE Consider a direct proof. Since n is even $\exists k \in \mathbb{Z}$

$\ni n = 2k$.

Notice $n+4 = 2k+4 = 2(k+2)$

which is even by definition.

T F There is a positive integer that equals the sum of the positive integers not exceeding it.

TRUE Consider 1. The positive integers not exceeding 1 are $\{1\}$

$1 = 1$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. (a) Write a proposition.

It will rain today.

(b) Write a propositional function P of x. *Hint*: clearly indicate your domain.

Don't forget this!

Domain: people at UWT

$P(x) :=$ x is registered for TCSS 321.

(c) Write a statement with $P(x)$, logical connectives, and quantifiers ($\forall$ or $\exists$) that evaluates to true.

$\exists x\ P(x)$

(d) Translate your answer for (c) into an English sentence.

There exists a person at UWT who is registered for UWT.

3. Find a compound proposition involving the propositional variables a, b, and c that is false when exactly two of a, b, and c are true, and true otherwise.

a	b	c	
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	F
T	F	F	T
T	F	T	F
T	T	F	F
T	T	T	T

so

$(a \wedge b \wedge c) \vee (a \wedge \neg b \wedge \neg c) \vee (\neg a \wedge b \wedge \neg c) \vee (\neg a \wedge \neg b \wedge c) \vee (\neg a \wedge \neg b \wedge \neg$

or

$\neg[(\neg a \wedge b \wedge c) \vee (a \wedge \neg b \wedge c) \vee (a \wedge b \wedge \neg c)]$

4. If you were promoted, then you bought the boss lunch.

(a) Write a different, but logically equivalent (English) statement as that given above.

It is necissary to buy the boss lunch to be promoted.
You bought the boss lunch if you were promoted.
If you did not buy the boss lunch, then you were not promoted. etc

(b) Given the above and that you bought the boss lunch, what conclusions (if any) can be drawn? Justify yourself.

No conclusions can be drawn.
Symbolically if p is "you were promoted" and q is "you bought the boss lunch" we were given $p \to q$ and q.

p	q	$p \to q$
F	F	T
F	T	T
T	F	F
T	T	T

5. *Hint*: Clearly define everything you use!!

   (a) Express the statement 'Nobody loves everybody'" using quantifiers, and logical connectives. Domain : all people Let $L(x,y)$ be "x loves y"

   There is no one who loves everybody
   There is no x such that x loves anybody
   $\neg[\exists x \, \forall y \, L(x,y)]$ or $\forall x \, \exists y \, \neg L(x,y)$

   (b) Negate part (a) so that any negation symbols immediately precede predicates.

   $\neg(\neg[\exists x \forall y \, L(x,y)])$
   $\exists x \, \forall y \, L(x,y)$

6. For the following "Theorem", determine:

   (a) if the "Proof" is valid, and
   (b) if the "Theorem" is true. If the "Theorem" is false, provide a counter example, and if the "Theorem" is true but the proof is not valid, provide a proof.

Theorem 1. *If n^2 is an even integer then n is even.*

Proof. Since n^2 is even there exists an integer a so that $n^2 = 2a$. Let $n = 2b$ for some integer b. This shows that n is even. □

Not valid: The proof assumed the conclusion

True.
We will prove the statement by using the contrapositive.
We assume that n is not even and will show
n^2 is not an even integer.
That is, we will assume n is odd and show n^2 is odd.

Since n is odd $\exists \, k \in \mathbb{Z} \ni n = 2k+1$.
Then $n^2 = (2k+1)^2 = (2k+1)(2k+1) = 4k^2 + 2k + 2k + 1$
$= 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$
Since $2k^2 + 2k$ is an integer, n^2 is of the form of an odd integer. Thus n^2 is odd & we've completed our proof. //

7. Given the following premises, what can you conclude about Hillary or David if anything? Justify yourself. "Allen is a bad boy or Hillary is a good girl." and "Allen is a good boy or David is happy".

[(Allen is a bad boy) v (Hillary is a good girl)]

AND [(Allen is a good boy) v (David is happy)]

$(\neg p \vee q) \wedge (p \vee r)$

$\Rightarrow q \vee r$

So Hillary is a good girl or David is happy.

8. You encounter two people, A and B on an island inhabited by only knights and knaves. The knights always tell the truth and knaves always lie. A says "The two of us are both knights" and B says "A is a knave." If possible, determine what these two people are and *justify* your conclusions.

There are four possibilities:

	A	B
1)	Knight	Knight
2)	Knight	Knave
3)	Knave	Knight
4)	Knave	Knave.

(1) is not possible b/c B would be lying but B is a Knight ~~to~~

(2) is not possible b/c A would be lying but A is a Knight ~~to~~

(4) is not possible b/c B would be saying the truth but B is a Knave

(3) contains no contradiction but it is the only remaining possibility.

Thus A is a Knave and B is a Knight.

9. Three logicians walk into a bar. The bar tender asks "Does everyone want a beer?". The first logician says, "I don't know". The second logician says, "I don't know". The third logician says, "Yes." Explain the joke.

Let B(x) = "x wants" a beer over the domain of the 3 logicians.

The bar tender asked if $\forall x\, B(x)$ or

B(logician one) $\wedge$ B(logician two) $\wedge$ B(logician three).

If logician one did not want a beer he could evaluate B(logician one) to F & thus the above statement to F.

However logician one could not evaluate the bartenders question to true without input from all the other logicians, thus his answer "I dont Know".

Similarly for logician 2.

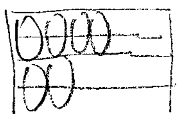
Logician 3 then had all the data & could evaluate the questio

10. Prove that you can use dominoes to tile a rectangular checkerboard with an even number of squares. We will construct a solution in a direct proof.

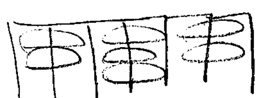
Let the checkerboard have m rows and n columns.
Then there are m·n squares on the checkerboard.

We were told m·n is even thus either m is even or n is even.

If m is even we can fit a whole # of dominoes in each column oriented vertically.

ex 

If n is even we can fit a whole # of dominoes in each row oriented horizontally

ex 

Thus either way we can tile the board //

11. Prove there are no rational number solutions to the equation $x^3 + x + 1 = 0$.

We will use proof by contradiction.
Assume $\exists$ a rational number that is a solution to $x^3+x+1=0$.

Then $\exists$ $a, b \in \mathbb{Z}$ where a & b are relatively prime and

$\left(\frac{a}{b}\right)^3 + \left(\frac{a}{b}\right) + 1 = 0$.

So $\frac{a^3}{b^3} + \frac{a}{b} = -1 \Rightarrow \frac{a^3+ab^2}{b^3} = -1 \Rightarrow a^3+ab^2 = -b^3$

$\Rightarrow a^3 = -b^3 - ab^2 \Rightarrow a^3 = b(-b^2-ab) \Rightarrow \frac{a^3}{b} = -b^2-ab$.

Notice $-b^2-ab$ is an integer so a^3/b is an integer, but that implies a and b are not relatively prime or that b=1. If a & b are not relatively prime, we have a contradiction! If b=1, then there is an integer solution to $x^3+x+1=0$ but this is equivalent to $x(x^2+1) = -1$ $\Rightarrow$ x has to be negative & no negative integers suffice //