Quiz 4

Key

TRUE/FALSE: Write "True" in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, write "False" and provide brief reasoning.

1. [2] (HW6 §4.1 #2) Let a, b, & c be positive integers & $a \neq 0$. If $a|(bc)$ then $a|b$ or $a|c$.

counterex (+1)
sense (+.5)

False (+.5)

Let $a=6$, $b=9$ and $c=4$
notice that $6|(9 \cdot 4)$ or $6|(36)$
but $6 \nmid 9$ and $6 \nmid 4$.

2. [2] (§9.1 #3d) The relation on the set $\{1,2,3,4\}$ defined by $\{(1,2),(2,1),(2,3),(3,4)\}$ is antisymmetric.

def (+.5)
counterex (+.5)
sense (+.5)

False (+.5)

Antisymmetric: If $(a,b) \in R$ and $(b,a) \in R$ then $a=b$ where R is a relation

Notice $(1,2)$ and $(2,1)$ are included but $1 \neq 2$.

3. [2] (Relations Wks #2) The relation on the set $\{1,2,3,4\}$ defined by $\{(a,b)|\max(a,b)=b\}$ is reflexive.

True (+.5)

Reflexive: If $\forall a \in \{1,2,3,4\}$ then (a,a) is in the relation.

def (+.5)
sense (+.5)
justification (+.5)

Notice $\max(a,a)=a$ so (a,a) is included for all $a \in \{1,2,3,4\}$

4. [3] (§9.5 #13) The relation on the set of all bit strings of length 3 or more defined by $\{(a,b)|$ the 1st and 3rd bits agree$\}$ is an equivalence relation.

justification (+1.5)
def (+1)

True (+.5)

An equivalence relation must be reflexive, symmetric, and transitive

Reflexive: Given a bit string a certainly the 1^{st} & 3^{rd} bits agree with itself so (a,a) is in the relation for all a ✓

Symmetric: If (a,b) is in the relation then the 1^{st} & 3^{rd} bits of a agree w/ the 1^{st} & 3^{rd} bits of b. So (b,a) is in the relation ✓

Transitive: If (a,b) & (b,c) are in the relation then the 1^{st} & 3^{rd} bits of a & b agree & also of b & c. So the 1^{st} & 3^{rd} bits of a & c match

Free Response: Show your work! No credit is given without supporting work.

5. Let R_{12} be a relation on $\mathbb{Z}$ defined as $\{(a,b) | a \equiv b \mod 12\}$

(a) [3] (HW6 §9.5 #1) Identify three numbers in the equivalency class of 4.

(+1.5) for #'s
eq class def (+.5)
sense (+.5)

$\ldots, -20, -8, 4, 16, 28, 40, \ldots$

(+.5)
note anything of the form $\{4 + k \cdot 12 \mid k \in \mathbb{Z}\}$

(b) [2] (Mod Wks) Compute $4 \mod 12 + 11 \mod 12$

$4 \text{ mod } 12 + 11 \text{ mod } 12 = 15 \text{ mod } 12$
$= 3 \text{ mod } 12$

note both work here

(c) [3] (HW6 §9.1 #2) Identify three ~~numbers~~ elements in the set $R_{12} \circ R_{12}$.

Know "$\circ$" (+1)
(+1.5) for elements
sense (+.5)

Some elements in R_{12}: $(0,0), (0,12), (0,24) \ldots (1,13), (1,-11), (1,1)$.
$(12,24) (12,0)$ $(13,1), (13,-11)$

So $0 \xrightarrow{R_{12}} 0 \xrightarrow{R_{12}} 0$
$\to 12 \to 12$

$\}$ $(0,0), (0,12), (12,12)$.
$(1,1), (1,13), (13,-11) \ldots$

really just R_{12} back again

(d) [3] Prove the equivalency class of 4 mod 12 intersected with the equivalency class of 0 mod 12 is empty.

intro (+.5) $\{$ Assume towards contradiction that $x \in \mathbb{Z}$
is in the equivalency class of 4 mod 12
& the equivalency class of 0 mod 12.

(+1) $\{$ Recall the equivalency class of 4 mod 12 $= [4]_{12} = \{4 + k12 \mid k \in \mathbb{Z}\}$
and the equivalency class of 0 mod 12 $= [0]_{12} = \{0 + t \cdot 12 \mid t \in \mathbb{Z}\}$

(+.5) $\{$ Since $x \in [4]_{12}$, $\exists k \in \mathbb{Z} \ni x = 4 + k \cdot 12 \Rightarrow 12 \nmid x$
Since $x \in [0]_{12}$ $\exists t \in \mathbb{Z} \ni x = 0 + t12 \Rightarrow 12 \mid x$

logic (+1) Contradiction!

So no x exists implying the intersection is empty.