EXAM 2 | TCSS 321 | PRACTICE

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, circle F and provide a counterexample or brief reasoning.

   T F $\{2, -4\} \in \{8.3, -4, 7.2, 4, 2, 0\}$.

   T F Let A and B be sets, then $A \setminus B = A \cap \overline{B}$.

   T F Let A, B, C, and D be sets, then $(A \times B) \times (C \times D) = A \times (B \times C) \times D$.

   T F The intersection of two uncountable sets cannot be finite.

   T F Let S be the set that contains a set x if the set x does not belong to itself. Then S is both a member and not a member of S

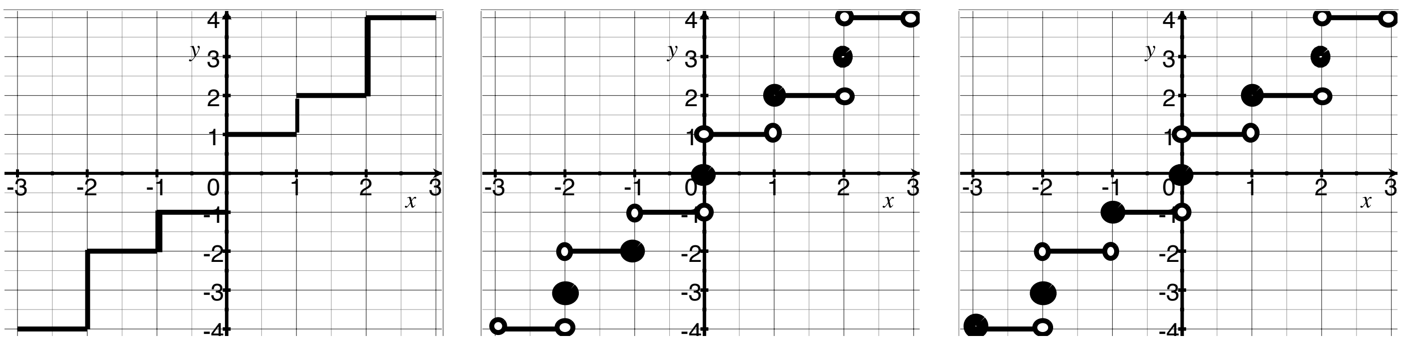
Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Let $f(x) = \lfloor x \rfloor + \lceil x/2 \rceil$ be a map from $\mathbb{R}$ to $\mathbb{R}$.

   (a) Evaluate $f(-2)$.

   (b) What is the range of f?

   (c) Which of the following is a graph of f? Briefly justify yourself.



3. Each of the following identify their complexity (big-O for programmers or big-Θ for computer scientists). Justify your conclusions as you would to a colleague.

   (a) $\dfrac{x^2+1}{x+1}$

   (b) $\sqrt{n} + \log n$

   (c) $1^3 + 2^3 + 3^3 + ... + n^3$

4. Consider the sequence $\{a\}$ defined recursively by $a_n = a_{n-1} + 2$ where $a_0 = 3$.

   (a) Find a_1 and a_2.

   (b) Find a (closed form) formula for a_n as a function of n (as opposed to using any terms previous to a_n in the formula).

5. Compute $\displaystyle\sum_{i=0}^{3}\left(\sum_{j=0}^{4}(2i+3j)\right)$.

6. Let $M = \begin{bmatrix} a & c \\ 0 & c \\ 0 & 0 \end{bmatrix}$, $N = \begin{bmatrix} 1 & 0 & b \\ 0 & f & a \end{bmatrix}$, and $P = \begin{bmatrix} 1 & 0 \\ 0 & f \\ f & 0 \end{bmatrix}$,
   where a, b, c, and f are nonzero real numbers. Find the following if possible:

   $M + P$ $\qquad\qquad$ $M^T + P$ $\qquad\qquad$ NP

7. Consider the algorithm described in pseudocode below for the next three questions.

```
Data: A, B: n × n matrices where n > 2
for i = 1 to n do
    for j = 1 to n do
        c_ij := 0
        if i ≤ j then
            for q := i to j do
                c_ij := c_ij + a_iq b_qj
            end
        end
    end
end
return C #C = [c_ij]
```

Algorithm 1: Matrix Algorithm

(a) Use Algorithm 1 by hand on $A = \begin{bmatrix} a & b \\ 0 & b \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$.
Clearly indicate your computations for each i and j.

(b) Describe what the algorithm is doing as you would to a colleague during lunch.

(c) Give a big-O estimate for the number of multiplications used in Algorithm 1 as a function of n. Show your reasoning.

8. Create and describe an algorithm for detecting if a string can be reordered into a palindrome.

9. Choose *ONE* of the following and support your conclusions. Clearly identify which of the two you are answering and what work you want to be considered for credit. Let A and B be sets.

   (a) Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ if and only if $A \subseteq B$.

   (b) Prove if $\mathcal{P}(A) = \mathcal{P}(B)$, then $A = B$.

10. Choose *ONE* of the following and support your conclusions. Clearly identify which of the two you are answering and what work you want to be considered for credit. Let A and B be sets.

   (a) Consider a rectangular checkerboard that you'd like to divide into its constituent squares. You can either make (complete) cuts along the vertical lines or horizontal lines separating the squares. Determine if there is a formula to determine how many cuts you must make to break the checkerboard into the n separate squares. If there is a formula, justify it. If there is no formula, prove that it does not exist.

   (b) For all positive integers n, $\sum_{i=1}^{n}(i^3) = \left(\frac{n(n+1)}{2}\right)^2$