EXAM 1 # TCSS 321 PRACTICE

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, circle F and provide a counterexample or brief reasoning. The logic symbols make use of the textbook notation.

   T F $\neg p \lor (r \to \neg q) \equiv \neg p \lor \neg q \lor r$

   T F Let n and m be integers. $\forall n \exists m (n^2 < m)$.

   T F Let $H(x)$ be "x is happy". Given the premise $\exists x H(x)$, we conclude that $H(\text{Lola})$. Thus, Lola is happy.

   T F If n is an even integer, than $n + 4$ is even.

   T F There is a positive integer that equals the sum of the positive integers not exceeding it.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. (a) Write a proposition.

   (b) Write a propositional function P of x. *Hint*: clearly indicate your domain.

   (c) Write a statement with $P(x)$, logical connectives, and quantifiers ($\forall$ or $\exists$) that evaluates to true.

   (d) Translate your answer for (c) into an English sentence.

3. Find a compound proposition involving the propositional variables a, b, and c that is false when exactly two of a, b, and c are true, and true otherwise.

4. If you were promoted, then you bought the boss lunch.

   (a) Write a different, but logically equivalent (English) statement as that given above.

   (b) Given the above and that you bought the boss lunch, what conclusions (if any) can be drawn? Justify yourself.

5. *Hint*: Clearly define everything you use!!

   (a) Express the statement 'Nobody loves everybody'" using quantifiers, and logical connectives.

   (b) Negate part (a) so that any negation symbols immediately precede predicates.

6. For the following "Theorem", determine:

   (a) if the "Proof" is valid, and
   (b) if the "Theorem" is true. If the "Theorem" is false, provide a counter example, and if the "Theorem" is true but the proof is not valid, provide a proof.

   Theorem 1. *If n^2 is an even integer then n is even.*

   Proof. Since n^2 is even there exists an integer a so that $n^2 = 2a$. Let $n = 2b$ for some integer b. This shows that n is even. □

7. Given the following premises, what can you conclude about Hillary or David if anything? Justify yourself. "Allen is a bad boy or Hillary is a good girl." and "Allen is a good boy or David is happy".

8. You encounter two people, A and B on an island inhabited by only knights and knaves. The knights always tell the truth and knaves always lie. A says "The two of us are both knights" and B says "A is a knave." If possible, determine what these two people are and *justify* your conclusions.

9. Three logicians walk into a bar. The bar tender asks "Does everyone want a beer?". The first logician says, "I don't know". The second logician says, "I don't know". The third logician says, "Yes." Explain the joke.

10. Prove that you can use dominoes to tile a rectangular checkerboard with an even number of squares.

11. Prove there are no rational number solutions to the equation $x^3 + x + 1 = 0$.