1. [3] Let $a, b$, and $c$ be integers, where $a \neq 0$. Prove that if $a \mid b$ and $b \mid c$, then $a \mid c$.
2. [3] Let $a, b$, and $c$ be positive integers and $a \neq 0$. Prove or disprove if $a \mid(b c)$ then $a \mid b$ or $a \mid c$.
3. [2] Let $a$ and $b$ be integers such that $a \equiv 11(\bmod 19)$ and $b \equiv 3(\bmod 19)$. Show work done by hand or Sage code used to find:
(a) the integer $c \in[0,18]$ such that $c \equiv 13 a(\bmod 19)$
(b) the integer $d \in[-18,0]$ such that $8 d \equiv 8 b(\bmod 19)$
4. [2] Show work done by hand or Sage code used to find the integer $a$ such that:
(a) $a \equiv 43(\bmod 23)$ and $-22 \leq a \leq 0$
(b) $a \equiv-1(\bmod 23)$ and $90 \leq a \leq 110$.
5. [3] Recall that congruence modulo 16 is an equivalence relation.
(a) Identify five numbers in the equivalence class of $-2 \bmod 16$.
(b) Identify five numbers in the equivalence class of $3 \bmod 16$.
(c) Use set builder notation to describe all the elements in the equivalence class of 3 $\bmod 16$.
6. [4] Let $R$ be the relation of logical equivalences on the set of all compound propositions.
(a) Show $R$ is an equivalence relation.
(b) Identify two elements that are in the equivalence class of $p \rightarrow q$.
7. [3] Define an equivalence relation on the set of classes offered at UWT.
(a) Justify why your relation is an equivalence relation.
(b) Determine the equivalence classes for your relation.
8. [3] Determine whether the relation $R$ on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if:
(a) $x+y=0$
(b) $x=2 y$
(c) $x=1$ or $y=1$
9. [5] Let $R_{5}$ and $R_{6}$ be the "congruent modulo 5 " and the "congruent modulo 6 " relations, respectively, on the set of integers. That is $R_{5}=\{(a, b) \mid a \equiv b(\bmod 5)\}$ and $R_{6}=\{(a, b) \mid a \equiv b(\bmod 6)\}$.
(a) Use quantifiers and "mod" to write out the set $R_{5} \cup R_{6}$.
(b) Identify three elements from the set $R_{6} \backslash R_{5}$
(c) Use quantifiers and "mod" to write out the set $R_{5} \oplus R_{6}$
(d) Identify three elements from the set $R_{5} \circ R_{5}$
(e) Identify three elements from the set $R_{5} \circ R_{6}$
10. [2] Define a relation on the set $A=\{0,1,2,5\}$ that is neither symmetric nor antisymmetric.
