EXAM 2 **TCSS 321** Spring 2015

1. [12] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, circle F and provide a counterexample or brief reasoning. The logic symbols make use of the textbook notation.

T F (§2.1 #17) Suppose A, B, and C are sets and that $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

T F (Quiz3 #2) The function e^n is $O(200,000n^2)$

T F (matrix wks #1) If M and N are matrices, then $MN = NM$

T F (HW3 §2.5 #2) The intersection of two countable infinite sets can never be countable infinite.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Let n be a positive integer. Consider the sets $S_3 = \{a_1, a_2, a_3\}$ and $S_n = \{a_1, a_2, a_3, ...a_n\}$.

   (a) [3] (set wks #5) Write down the elements of $\mathcal{P}(S_3)$.

   (b) [2] (set wks #6) Find $|\mathcal{P}(S_n)|$.

3. (§2.4 #5) Consider the function f that assigns to each bit string the number of ones in the string minus the number of zeros in the string.

   (a) [2] Evaluate $f(01110111)$.

   (b) [2] Identify the range of f.

4. [5] (§3.1 #31) Devise an algorithms that finds the first term of a sequence of integers that equals some previous term in the sequence.

5. [3] (HW5 §2.6 #4) Give a big-O estimate for the number of comparisons between integers that your algorithm in the previous problem uses. Justify your answer.

6. Consider the MysterySearch code shown to the right.

   (a) [3] (bigOintro wks#4) Give a big-O estimate for the number of comparisons between integers that the algorithms makes. Justify your answer.

```
def MysterySearch(x, list):
    i=0
    while i<=len(list)-1 and x!=list[i]:
        i=i+1
    if i<=len(list)-1:
        location=i
    else:
        location=-1
    return location
```

```
MysterySearch(4,[1,2,3,5,6,7,8,11,12,14,10])
-1
```

```
MysterySearch(11,[1,2,3,5,6,7,8,11,12,14,10])
7
```

   (b) [2] (Quiz 3 #6) How does the number of comparisons change in MysterySearch when the inputs change from n to n^2.

   (c) [2] (Quiz 3 #6) Recall that Binary Search is $O(\log n)$. How does the number of comparisons change in MysterySearch when the inputs change from n to n^2.

7. [4] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
   No, doing both questions will not earn you extra credit.

   (a) (§2.4 #11) Let $a_n = 2^n + 5 \cdot 3^n$ for $n = 0, 1, 2 \ldots$. Prove that $a_n = 5a_{n-1} - 6a_{n-2}$ for all n greater than or equal to two.

   (b) (§1.8 #5) Prove that $\min(x, y) = \frac{x+y-|x-y|}{2}$ whenever x and y are real numbers.

8. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
   No, doing both questions will not earn you extra credit.

   (a) (§5.1 #5) Prove that whenever n is a positive integer that,

   $$\sum_{i=0}^{n}(2i+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$$

   (b) (pg 379 #4) Prove that whenever n is a positive integer that,

   $$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\ldots\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$$