EXAM 1 | TCSS 321 | Winter 2015

1. [12] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, circle F and provide a counterexample or brief reasoning. The logic symbols make use of the textbook notation.

   T F (Logic2Wks #2) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

   T F (HW2 §1.5 #3) Given the domain of real numbers, $\forall x\, \exists y, (x = y^2)$

   T F (EC ProofWks) If a shape with a checkerboard pattern is tillable with dominoes, then the shape has the same number of white and black spaces.
   Note: each domino covers two squares on a checkerboard patterned shape.

   T F (EC ProofWks) If a shape with a checkerboard pattern has the same number of white and black spaces, then the shape is tillable with dominoes.
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Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [2] (PracticeExam #2) Write a proposition (in English).

3. Consider the following: "If it snows today, then I will ski tomorrow."

   (a) [3] (§1.1 #27) Write a different, but logically equivalent (English) statement as that given above.

   (b) [2] (§1.6 #9) Given the above and that "I will not ski tomorrow.", what conclusions (if any) can be drawn? Justify yourself.

4. Consider the SAGE code to the right.

   (a) [2] (SageDemo) Explain what the code is doing in english.

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a=[-10,..10]

answer=-10+2==5

for x in a:
    if x+2==5:
        answer=answer&True
    else:
        answer=answer&False
print answer
 False
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   (b) [3] (HW2 §1.5 #2) Use predicates, quantifiers, logical connectives, and mathematical operators to express what the Sage code is doing.

5. (HW1 §1.4 #5) Consider the statement "Nobody can fool me."

   (a) [4] Express the statement above using quantifiers and logical connectives.

   (b) [3] Form the negation of part (a) so that no negation appears to the left of a quantifier.

6. [3] (Quiz1 #2) Find a compound proposition involving the propositional variables a, b, and c that is true when a and b are true and c is false, but false otherwise.

7. For the following "Theorem", determine:

   (a) [2] if the "Proof" is valid. If the proof is not valid, identify the error, and
   (b) [3] if the "Theorem" is true. If the "Theorem" is false, provide a counter example, and if the "Theorem" is true but the proof is not valid, provide a proof.

   Theorem 1. *If n^2 is positive, then n is positive.*

   Proof. Assume that n^2 is positive. We want to show that n is positive.

   Consider if n is positive, then $n > 0$. We can multiply this inequality by the positive number n on both sides and arrive at $n^2 > 0n$ or $n^2 > 0$. Thus, if n is positive we know n^2 is positive.

   Since we were assuming that n^2 was positive, we can conclude from the above that n is positive. □

8. [4] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
   No, doing both questions will not earn you extra credit.

   (a) (§1.2 #21) Suppose you are on an island that has two kinds of inhabitants, knights who always tell the truth, and their opposites, knaves who always lie. You encounter two people A and B. What are A and B if A says "I am a knave or B is a knight." and B says nothing. If possible, determine what these two people are and justify your conclusions.

   (b) (§1.2 #15) Each inhabitant of a remote village always tells the truth or always lies. A villager will give only a "Yes" or a "No" response to a question a tourist asks. Suppose you are a tourist visiting this area and come to a form in the road. One branch leads to the ruins you want to visit; the other branch leads deem into the jungle. A villager is standing at the form in the road. What one question can you ask the villager to determine which branch to take? Explain why your question will determine which road is which.

9. [6] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
   No, doing both questions will not earn you extra credit.

   (a) (EC Wks #2) Let m and n be integers. If mn is even, then m is even or n is even.

   (b) (EC Wks #12) Prove there exists no positive integer n such that $n^2 + n^3 = 100$.