Key name (+1)

EXAM 1 TCSS 321 ~~Fall 2012~~ Winter 2018

1. [12] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, circle F and provide a counterexample or brief reasoning. The logic symbols make use of the textbook notation.

true (+.5)
start reasoning (+.5)
logic/justify (+1.5)
sense (+.5)

T F (Logic2Wks #2) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
F	F	F	F	F
F	F	T	F	F
F	T	F	F	F
F	T	T	F	F
T	F	F	F	F
T	F	T	T	T
T	T	F	T	T
T	T	T	T	T

note: both have the same truth table

T F (HW2 §1.5 #3) Given the domain of real numbers, $\forall x \, \exists y, (x = y^2)$

false (+.5)
start reasoning (+.5)
logic/justify (+1.5)
sense (+.5)

for all real numbers x there exists a y such that $x = y^2$

Consider $-1 = x$

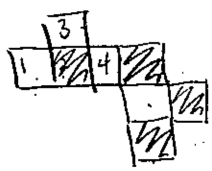
T F (EC ProofWks) If a shape with a checkerboard pattern is tillable with dominoes, then the shape has the same number of white and black spaces.
Note: each domino covers two squares on a checkerboard patterned shape.

true (+.5)
start reasoning (+.5)
logic/justify (+1.5)
sense (+.5)

Each domino covers one white & one black square on a checkerboard patterned shape. Thus each domino placed will add one to both the total number of white spots & total number of black spots covered.

T F (EC ProofWks) If a shape with a checkerboard pattern has the same number of white and black spaces, then the shape is tillable with dominoes.
Note: each domino covers two squares on a checkerboard patterned shape.

false (+.5)
start reasoning (+.5)
logic/justify (+1.5)
sense (+.5)



Notice a domino that is covering space 2 can cover space 1 xor space 3.

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Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [2] (PracticeExam #2) Write a proposition (in English).

start (+.5)
sentence (+.5)
true/false (+1)

A statement that is true xor false.
(it cannot be both?)

3. Consider the following: "If it snows today, then I will ski tomorrow."

   (a) [3] (§1.1 #27) Write a different, but logically equivalent (English) statement as that given above.

(contrapositive)
start (+.5)
equivalent (+2)
sense/notation (+.5)

If I don't ski tomorrow then it did not snow today
You will ski tomorrow, if it snows today

Note: There are many correct answers here?

   (b) [2] (§1.6 #9) Given the above and that "I will not ski tomorrow.", what conclusions (if any) can be drawn? Justify yourself.

(+1) { We can conclude that it will not snow today
(+1) { using the contrapositive of the statement in (a).

4. Consider the SAGE code to the right.

```
a=[-10,..10]

answer=-10+2==5

for x in a:
    if x+2==5:
        answer=answer&True
    else:
        answer=answer&False
print answer
 False
```

   (a) [2] (SageDemo) Explain what the code is doing in english.

start (+.5)
(+.5) {
(+.5) {
(+.5) {

Repeated and's?
Is checking if for all $x \in \mathbb{Z}$ between -10 and 10 if $x+2=5$.

   (b) [3] (HW2 §1.5 #2) Use predicates, quantifiers, logical connectives, and mathematical operators to express what the Sage code is doing.

(+1) { Domain $x \in \{-10, -9, -8, -7, \ldots 8, 9, 10\}$

$\underbrace{\forall x}_{(+1)} \quad \underbrace{x+2=5}_{(+1)}$.

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5. (HW1 §1.4 #5) Consider the statement "Nobody can fool me."

   (a) [4] Express the statement above using quantifiers and logical connectives.

   (+1) Domain: people work (+.5) No person can fool me

   (+1) $F(x)$: x can fool me (There exists no person who can fool me

   $$\nexists x\, F(x) \equiv \forall x\, \neg F(x)$$

   got it (+1) sense/notation (+.5)

   (b) [3] Form the negation of part (a) so that no negation appears to the left of a quantifier.

   $$\neg(\nexists x\, F(x)) \equiv \exists x\, F(x)$$

   (+1) notation/sense (+.5)

   move to the left of quantifiers (+.5)

   switch $\forall$ & $\exists$'s (+.5)

   negated propositional functions (+.5)

6. [3] (Quiz1 #2) Find a compound proposition involving the propositional variables a, b, and c that is true when a and b are true and c is false, but false otherwise.

a	b	c	
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	F
T	F	F	F
T	F	T	F
T	T	F	T
T	T	T	F

understood what wanted (+1)

$a \wedge b \wedge \neg c$

consistent w/ work (+1)

got it (+1)

7. For the following "Theorem", determine:

   (a) [2] if the "Proof" is valid. If the proof is not valid, identify the error, and

   (b) [3] if the "Theorem" is true. If the "Theorem" is false, provide a counter example, and if the "Theorem" is true but the proof is not valid, provide a proof.

Theorem 1. *If n^2 is positive, then n is positive.*

Proof. Assume that n^2 is positive. We want to show that n is positive.

Consider if n is positive, then $n > 0$. We can multiply this inequality by the positive number n on both sides and arrive at $n^2 > 0n$ or $n^2 > 0$. Thus, if n is positive we know n^2 is positive.

Since we were assuming that n^2 was positive, we can conclude from the above that n is positive. □

(+.5) (+1) (+.5) start

a) Not valid. The proof showed the converse of the Theorem's statement.

(+1)

b) Not true. Notice $(-2)^2 = 4 > 0$ but $-2 \not> 0$.

counter ex (+1)

start (+.5)

notation/sense (+.5)

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8. [4] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (§1.2 #21) Suppose you are on an island that has two kinds of inhabitants, knights who always tell the truth, and their opposites, knaves who always lie. You encounter two people A and B. What are A and B if A says "I am a knave or B is a knight." and B says nothing. If possible, determine what these two people are and justify your conclusions.

(b) (§1.2 #15) Each inhabitant of a remote village always tells the truth or always lies. A villager will give only a "Yes" or a "No" response to a question a tourist asks. Suppose you are a tourist visiting this area and come to a form in the road. One branch leads to the ruins you want to visit; the other branch leads deem into the jungle. A villager is standing at the form in the road. What one question can you ask the villager to determine which branch to take? Explain why your question will determine which road is which.

Start (+.5)
Consider all cases (+1)
reasoning logic (+1)
sense (+1)
got it (+.5)

(a) There are four possibilities:

A	Knight	Knight	Knave	Knave
B	Knight	Knave	Knight	Knave
		A would by lying but A is a knight	A would be telling the truth but A is a knave	

the only possibility!

(b) "If I asked you if the road on the right lead to the ruins, what would you say?"

road to the right	ruins	ruins	jungle	jungle
villager type	truth	lie	truth	lie
answer given:	Yes	Yes	No	No

9. [6] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

Start +1
intro/plan +1
define terms +1
logic +1.5
clarity +1
style/notation +.5

(a) (EC Wks #2) Let m and n be integers. If mn is even, then m is even or n is even.

(b) (EC Wks #12) Prove there exists no positive integer n such that $n^2 + n^3 = 100$.

(a) We will show the contrapositive. That is assume m AND n are odd, we will show that $m \cdot n$ is odd.

Since m and n are odd, $\exists\, l, k \in \mathbb{Z}$ so that $m = 2l+1$ and $n = 2k+1$.

Then $m \cdot n = (2l+1)(2k+1)$
$= 4lk + 2l + 2k + 1$
$= 2(2lk + l + k) + 1$

which is odd. Thus $m \cdot n$ is odd, which is what we wanted to s...

(b) We will exhaustively show there is no positive integers n such that $n^2 + n^3 = 100$.

Notice $n^2 + n^3 = 100 \iff n^2(1+n) = 100$.

Consider $n=1$, $(1)^2(1+1) \neq 100$,
$n=2$, $2^2(1+2) \neq 100$,
$n=3$, $3^2(1+3) \neq 100$,
$n=4$, $4^2(1+4) \neq 100$
$n=5$, $5^2(1+5) = 150 > 100$

As n increases $n^2(1+n)$ will too, so no $n > 5$ will satisfy $n^2(1+n) = 100$.
Thus no integer n exists.

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