Use set builder notation and logical equivalences to establish the first De Morgan law $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

Solution: We can prove this identity with the following steps.

$$
\begin{aligned}
\overline{A \cap B} &= \{x \mid x \notin A \cap B\} & &\text{by definition of complement} \\
&= \{x \mid \neg(x \in (A \cap B))\} & &\text{by definition of does not belong symbol} \\
&= \{x \mid \neg(x \in A \wedge x \in B)\} & &\text{by definition of intersection} \\
&= \{x \mid \neg(x \in A) \vee \neg(x \in B)\} & &\text{by the first De Morgan law for logical equivalences} \\
&= \{x \mid x \notin A \vee x \notin B\} & &\text{by definition of does not belong symbol} \\
&= \{x \mid x \in \overline{A} \vee x \in \overline{B}\} & &\text{by definition of complement} \\
&= \{x \mid x \in \overline{A} \cup \overline{B}\} & &\text{by definition of union} \\
&= \overline{A} \cup \overline{B} & &\text{by meaning of set builder notation}
\end{aligned}
$$

Note that besides the definitions of complement, union, set membership, and set builder notation, this proof uses the second De Morgan law for logical equivalences. ◄