

Typo.

Relations

R_1 is a relation on \mathbb{Z} defined by $\{(a, b) | a + b \leq 3\}$

R_2 is a relation from $A = \{1, 2, 3, 4\}$ to $B = \{1, 2, 3\}$ defined by $\{(a, b) | \gcd(a, b) = 1\}$

R_3 is a relation on \mathbb{Z} defined by $\{(a, b) | a = b^2\}$

R_4 is a relation on \mathbb{R} defined by $\{(a, b) | a \cdot b \geq 0\}$

1. For each of the relations R_i defined above, list five ordered pairs that are in the relation.

R_1 :

R_2 :

R_3 :

R_4 :

2. For each of the relations R_i above, determine if R_i has the properties listed (if applicable). If the relation does not have the indicated property, identify an ordered pair (or set of ordered pairs) that exhibits the failure.

	R_1	R_2	R_3	R_4
reflexive	no 2, 2	n/a	no -1, -1	yes
symmetric	yes completely	n/a	no 1, 2 -1, 1	yes
antisymmetric	no OR 3	n/a	yes	no
transitive	no 2, 2 1, 0 OR 3 OR 2, 3	n/a	no	no -1, 2 OR 7 -1, 7

3. Find an example of a relation on a set that is both symmetric and antisymmetric.

Let $A = \{ \emptyset, \{1\}, \{2\}, \{1,2\} \}$ be the powerset on 2 elements
 $R = \{ (\emptyset, \emptyset), (\{1\}, \{1\}), (\{2\}, \{2\}), (\{1,2\}, \{1,2\}) \}$

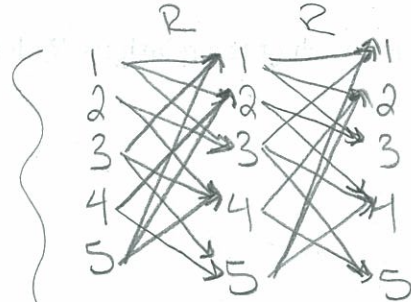
Note $\forall a \in A$ if $a R b$ then $b R a$ (symmetric)
 and if $a R b$ and $b R a$ then $b = a$. (antisymmetric)

4. Let R be the relation on the set $\{1, 2, 3, 4, 5\}$ containing the ordered pairs:

$(1, 1), (1, 2), (1, 3), (2, 3), (2, 4), (3, 1), (3, 4), (3, 5), (4, 2), (4, 5), (5, 1), (5, 2),$ and $(5, 4)$.

- (a) Find $R \circ R$

$\left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5) \\ (2,1), (2,4), (2,5), (2,2) \\ (3,1), (3,2), (3,3), (3,5) \\ (4,3), (4,4), (4,1), (4,2) \\ (5,1), (5,2), (5,3), (5,4), (5,5) \end{array} \right\}$



- (b) Find R^2

same as above b/c
 $R^2 = R \circ R$

- (c) Find R^3

$\left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5) \\ (2,1), (2,2), (2,3), (2,4) \\ (3,1), (3,2), (3,3), (3,4), (3,5) \\ (4,1), (4,2), (4,3), (4,4), (4,5) \\ (5,1), (5,2), (5,3), (5,4), (5,5) \end{array} \right\}$

