Quiz 4

Show *all* your work. No credit is given without reasonable supporting work. There are *two* sides to this quiz and all logic symbols make use of the textbook notation.

1. [2] The "structure" of induction requires two steps. Name the two steps.

2. [3] (§5.1 Example 15) Determine if the following proof is valid or not. If valid, identify the proof method(s) used. If not valid, highlight the fallacies/error in logic.

   Theorem 1. *Every set of lines in the plane, no two of which are parallel, meet in a common point.*

   Proof 1. *Certainly two lines that are not parallel meet in a common point.*

   Assume a collection of k lines, no two of which are parallel, meet at a common point. We will show that a collection of $k+1$ lines, no two of which are parallel, will also meet at a common point.

   Consider the set of $k+1$ distinct lines in the plane. By our hypothesis, the first k of these lines meet in a common point p_1. Similarly the last k of these lines meet in a common point p_2. We will show that p_1 and p_2 must be the same point.

   If p_1 and p_2 were different points, all lines containing both of them must be the same line because two points determine a line. This contradicts our assumption that all these lines are distinct. Thus, p_1 and p_2 are the same point. We conclude that the point $p_1 = p_2$ lies on all $k+1$ lines, thus our $k+1$ distinct lines meet at a common point.

3. [3] Explain how inductions works as you would to a colleague who "doesn't believe" in induction.

4. [6] (§5.2 #10) Assume that a chocolate bar consists of n squares arranged in a rectangular pattern. The entire bar, a smaller rectangular piece of the bar, can be broken along a vertical of a horizontal line separating the squares. Assume that only one piece can be broken at a time. Prove you will need $n-1$ breaks to break the bar into n separate squares.

5. [6] (§5.1 #7) Prove that $3 + 3 \cdot 5 + 3 \cdot 5^2 + 3 \cdot 5^3 + \cdots + 3 \cdot 5^n = \dfrac{3(5^{n+1} - 1)}{4}$ whenever n is a nonnegative integer.