Quiz 2

Show *all* your work. No credit is given without reasonable supporting work. There are *two* sides to this quiz and all logic symbols make use of the textbook notation.

1. [4] (§1.6 #9) Consider the premises given below. What conclusion or conclusions can be drawn? Explain/justify your reasoning.

   - "Every computer science major has a personal computer."
   - "Ralph does not have a personal computer."
   - "Ann has a personal computer."

2. (logic con't wks) Read the following "Theorem". Determine (and justify):

   (a) [2] if the "theorem" is true, and
   (b) [4] if the "proof" is valid.

   Theorem 1. *Let m, n, and p be integers. If $m+n$ and $n+p$ are even integers, then $m+p$ is an even integer.*

   Proof. Assume that $m+n$ and $n+p$ are even integers. We want to show that $m+p$ is an even integer.

   Since $m+n$ is an even integer, there exists integers a and b such that

   $$m+n=2a+2b.$$

   Thus we know that $m=2a$ and $n=2b$.

   Since $n+p$ is an even integer and $n=2b$, we know that there exists and integer c such that

   $$n+p=2b+2c.$$

   Thus we also know that $p=2c$.

   Now we can consider $m+p$ which equals $2a+2c=2(a+c)$. Thus $m+p$ is even. $\square$

3. (§1.5 #10) [4] Express the statement "There is no one who can fool everybody" using predicates, quantifiers, and logical connectives.

4. (§1.7) [6] Prove *one* of the following. Only one proof will be graded so make sure that you clearly identify the work you want considered.

   (a) (3/3 lecture) If a and b are real numbers with $a \neq 0$, then there is a unique real number r such that $ar + b = 0$.

   (b) (§1.7 #27) Prove that if n is a positive integer, then n is odd if and only if $5n + 6$ is odd.