Proofs

the mostly algebraic kind

1. For each of the following "Theorems", determine if the argument is valid.

Theorem 1. *If n is an odd integer, then n^2 is odd.*

Proof. Assume that n is an odd integer. We want to show that n^2 is odd.

Since n is an odd integer, there exists an integer a, so that $n = 2a + 1$. Using algebra we see,

$$n^2 = (2a + 1)^2 = 4a^2 + 4a + 1 = 2(2a^2 + 2a) + 1.$$

By definition of an odd integer we see n^2 is an odd integer. □

Theorem 2. *If n^2 is positive, then n is positive.*

Proof. Assume that n^2 is positive. We want to show that n is positive.

Consider if n is positive, then $n > 0$. We can multiply this inequality by the positive number n on both sides and arrive at $n^2 > 0n$ or $n^2 > 0$. Thus, if n is positive we know n^2 is positive.

Since we were assuming that n^2 was positive, we can conclude from the above that n is positive. □

Theorem 3. *n is an even integer if and only if n^2 is an even integer.*

Proof. Assume that n is an even integer. We want to show that n^2 is an even integer.

Since n is an even integer, there exists and integer a such that $n = 2a$. Using algebra we see,

$$n^2 = (2a)^2 = 4a^2 = 2(2a^2)$$

. By definition of an even integer we see n^2 is an even integer. □

Theorem 4. *If n is an integer and n^2 is odd, then n is odd.*

Proof. We will show the contrapositive of the statement, that is, if n is not odd, we will show that either n is not an integer or n^2 is not odd. Said more simply, we will show, if n is even, then either n wasn't an integer or n^2 is even.

Since n is even, we know there exists an integer a such that $n = 2a$. Using algebra we see,

$$n^2 = (2a)^2 = 4a^2 = 2(2a^2),$$

thus, by definition of an even integer, n^2 is even. □

Consider verifying your answers by looking at §1.7 Examples 1, 16, and 8.

2. (§1.7 #15) Let x and y be real numbers. Prove that if $x + y \geq 2$, then $x \geq 1$ or $y \geq 1$.

3. (§1.7 #9) Prove that the sum of an irrational number and a rational number is irrational.

4. (§1.8 Ex5) Prove the final decimal digit of the square of an integer is 0, 1, 4, 5, 6, or 9.