Somemore Logic

with quantifiers!

Let p, q, and r be propositions for the entirety of this worksheet (front & back).

1. Below is all the ways I could think to combing (using textbook notation) $\neg$, $\wedge$, and $\vee$.

   (a) Find the truth tables for each.

(i) $\neg(p \wedge q)$ (ii) $(\neg p) \wedge q$ (iii) $p \wedge (\neg q)$ (iv) $(\neg p) \wedge (\neg q)$

(v) $\neg(p \vee q)$ (vi) $(\neg p) \vee q$ (vii) $p \vee (\neg q)$ (viii) $(\neg p) \vee (\neg q)$

   (b) Do any of the above truth tables look the same (i.e. are there any logical equivalences)? Which ones?

2. Perform the same investigation on the following (using textbook notation) and try to identify another logical equivalence.

(i) $p \vee (q \wedge r)$ (ii) $p \wedge (q \vee r)$ (iii) $(p \vee q) \wedge (p \vee r)$ (iv) $(p \wedge q) \vee (p \wedge r)$

Let x be a real number in the following.

3. Translate the following symbolic propositions into English sentences and determine the truth value.

   (a) $\forall x, x + 1 > x$.

   (b) $\exists x, x + 1 > x$.

   (c) $\forall x < 0, x^2 > 0$.

   (d) $\forall x, (x < 0 \rightarrow x^2 > 0)$

   (e) $\exists x > 0, x^2 = 2$.

   (f) $\exists x, (x > 0 \wedge x^2 = 2)$.

   (g) $\neg \forall x, x^2 > x$.

   (h) $\exists x, \neg(x^2 > x)$.

4. Do any of the above statements seem logically equivalent? Which ones?

Verify your answers by examining Examples 8,10,17, &21 of §1.4.