EXAM 2 | TCSS 321 | Fall 2012

1. [6] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, circle F and provide a counterexample or brief reasoning.

   T F Binary search can locate a specified element, if it exists, in an arbitrary sequence.

   T F If $ac \equiv bc \mod m$, where a, b, c, and m are integers with $m \geq 2$, then $a \equiv b \mod m$.

   T F There are a countable infinite number of primes.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. [2] What's your favorite algorithm? Why?

3. [4] (§4.3 #31) The product of two integers is $2^5 \cdot 3^8 \cdot 5^3 \cdot 7^{11}$ and their greatest common divisor is $2 \cdot 3^7 \cdot 7^9$. What is their least common multiple? Show your work or your reasoning.

4. Consider the Sort algorithm described below for the next three questions.

Data: a_0, a_1, ... a_{n-1}: integers where $n > 2$
Result: integers sorted in increasing order

for $i = 0$ *to* $n-1$ **do**
    smallSub=i;
    for $j = i$ *to* $n-1$ **do**
        if $a_j < a_{smallSub}$ **then**
            smallSub=j;
        end
        temp $=a_i$;
        $a_i = a_{\text{smallSub}}$;
        a_{smallSub} =temp;
    end
end

Algorithm 1: Sort Algorithm

(a) [4] (Quiz3 #4) List the steps used by the Sort algorithm on the input $\{7, 6, 8, 5\}$. Clearly indicate when and what swaps/recordings/interchanges are made.

(b) [2] Describe the Sort algorithm as you would to a colleague during lunch.

(c) [2] (§3.3 #3) Give a big-O estimate for the comparisons used in the sort algorithm. Show your reasoning.

5. Each of the following give an exact count of operations taken in a worse case senerio for a certain algorithm. Identify their complexity (big-O for programmers or big-Θ for computer scientists) as you would to a colleague and justify yourself.

   (a) [2] (§3.2 #6) $\dfrac{x^3 + 2x}{2x + 1}$

   (b) [2] (§3.2 #7b) $4x^3 + (\log x)^4$

   (c) [2] (§3.2 #19) $2^{x+1} + 8x^{15}$

6. [5] Choose *ONE* of the following and support your conclusions. Clearly identify which of the two you are answering and what work you want to be considered for credit.

   (a) (§3.1 #23) Clearly describe an algorithm that determine whether a function from a finite set of integers to another finite set of integers is onto. Specifically the inputs are: f:function, $[x_0, x_1, ...x_n]$:integers in domain, $[y_0, y_1, ...y_m]$: integers in codomain.

   (b) (§3.1 #32) Clearly describe and algorithm that finds all terms of a finite sequence of integers that are greater then the sum of all previous terms in sequence.

7. (a) [3] (§4.2 #21) Find $10011_2 + 10110_2$

(b) [2] (§4.2 #3) Convert the binary number 10110_2 into decimal.

(c) [3] (§4.2 #40) Recall the two's complement representation of an integer:
To represent an integer x between $2^{n-1} \leq x \leq 2^{n-1}$ using n bits we use the leftmost bit to record the 'sign'. A 0 bit in this position is used for a positive integer and a 1 bit in this position is used for a negative integer. For a positive integer, the remaining bits are identical to the binary expansion of the integer. For a negative integer, the remaining bits are the bits of the binary expansion of $2^{n-1} - |x|$.
Find the 6 bit two's complement to -21.

(d) [2] (§4.2 #3) Consider the answer to (c) as only a binary number (as opposed to a two's complement representation). Convert the above binary number into decimal.

(e) [3] (§4.1 #13) Find c such that $c + 21 \equiv 0 \bmod 32$.

(f) [$ExtraCredit$] Identify how the two's complement representation with 6 bits is related to arithmetic modulo 2^6 or $\mathbb{Z}_{32}$.

8. [6] Choose *ONE* of the following and support your conclusions. Clearly identify which of the two you are answering and what work you want to be considered for credit.

   (a) (§4.1 #7) Prove if a, b, and c are positive integers where $a \neq 0$ and $c \neq 0$ such that $(ac)|(bc)$ then $a|b$.

   (b) (Lecture 11/5) Prove if a, b, and c are positive integers such that $\gcd(a, b) = 1$ and $a|(bc)$, then $a|c$.