EXAM 1 | TCSS 321 | Fall 2012

1. [8] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true and briefly justify your answer. Otherwise, circle F and provide a counterexample or brief reasoning. The logic symbols make use of the textbook notation.

   T F If $p =$ True, and $q =$ True, then $(p \vee q) \vee \neg(p \wedge q) =$ True.

   T F $\neg\forall x, \exists y, [P(x,y) \wedge Q(y,x)] = \exists x, \forall y, [\neg P(x,y) \vee \neg Q(y,x)]$

   T F $\displaystyle\sum_{i=0}^{3}\sum_{j=1}^{2}(i-j) = 3$

   T F There are more rational numbers than real numbers between 0 and 1.

Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

2. Let the domain be students at the University of Washington, Tacoma.

   (a) [2] (Quiz1 #1) Write a proposition.

   (b) [3] (§1.4 Ex1) Write a propositional function P of x where x is in your domain.

3. Consider the following: "If there is going to be a quiz, then I will come to class."

   (a) [3] (HW §1.1 #28) Write a different, but logically equivalent (English) statement as that given above.

   (b) [2] (proof intro wks #1) Given the above and that "There will be no quiz.", what conclusions (if any) can be drawn? Justify yourself.

4. (Quiz1 #3) Consider the following statement: "There is a pig that can spin spider webs and talk."

   (a) [3] Express the statement above using predicates, quantifiers, and logical connectives.

   (b) [2] Negate part (a) so that the negation is to the left of any quantifiers.

5. [4] (§2.4 Ex) Find a recurrence relation satisfied by the sequence: $a_n = 2 + 3(n - 1)$.

6. [3] (function wks #5) Create a function that is a surjection/onto but not one-to-one. Justify your decision.

7. (§2.5 #11) For each of the following find two uncountable sets A and B so that:

   (a) [2] $A \cap B$ is finite

   (b) [2] $A \setminus B$ is countable infinite.

8. [4] Choose *ONE* of the following and support your conclusions. Clearly identify which of the two you are answering and what work you want to be considered for credit.

   (a) (§1.2 #21) Suppose you are on an island that has two kinds of inhabitants, knights who always tell the truth, and their opposites, knaves who always lie. You encounter two people A and B. What are A and B if A says "I am a knave or B is a knight." and B says nothing.

   (b) (cardinality wks) Hilbert is running a hotel with a countably infinite number of rooms, all of which are occupied. Hilbert's business is doing so well that the hotel expands to a second building which also contains a countably infinite number of rooms (initially unoccupied). Can Hilbert rearrange his guests so his rooms are still all occupied?

9. (§2.6 #1&3) Let $N = \begin{bmatrix} 0 & 0 & c \\ 0 & b & -c \\ a & a & 0 \end{bmatrix}$, $P = \begin{bmatrix} 1 & 0 & a \\ 0 & f & a \end{bmatrix}$, and $Q = \begin{bmatrix} 1 & 0 \\ 0 & f \\ f & 0 \end{bmatrix}$, where a, b, c, and f are nonzero real numbers. Find the following if possible:

(a) [2] $P + Q^{\mathrm{T}}$ (b) [2] NP (c) [2] PQ

10. [6] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

    (a) (HW §1.7) Prove that the product of a nonzero rational number and an irrational number is irrational.

    (b) (§1.7 Ex4) Prove that if $n = ab$, where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$.