

# Equivalence Relations

1. Let  $R$  be a relation on  $\mathbb{Z}$  defined by  $\{(a, b) | a \neq b\}$

(a) Identify five ordered pairs in  $R$ .

(b) What is the reflexive closure of  $R$ ?

(c) What is the symmetric closure of  $R$ ?

(d) What is the antisymmetric closure of  $R$ ?

2. Let  $T = \{(1, 3), (1, 4), (2, 1), (3, 2)\}$  be the relation on the set  $\{1, 2, 3, 4\}$ .

(a) What is the reflexive closure of  $T$ ?

(b) What is the symmetric closure of  $T$ ?

(c) What is the transitive closure of  $T$ ?

3. For each of the following determine if the relation defines an equivalence relation and if it does, identify the equivalence classes.

(a) Let  $R$  be the relation on the set of all triangles defined by  $\{(\triangle abc, \triangle def) \mid \angle a = \angle d \text{ and } \angle b = \angle e\}$

(b) Let  $S$  be a relation on the set of all functions from  $\mathbb{Z}$  to  $\mathbb{Z}$  defined by  $\{(f, g) \mid f(0) = g(0) \text{ and } f(1) = g(1)\}$ .

(c) Let  $T$  be a relation on the set of all sets of real numbers which that  $A \sim B$  if and only if  $|A| = |B|$ .

(d) Let  $U$  be a relation on the set  $\{a, b, c, d\}$  with the corresponding directed graph.

