

Cardinality

the infinite kind

David Hilbert was a great mathematician (whose bio is on page 171) who single handedly steered 100 years of mathematical work by asking 23 questions in 1900.

But more to the point of the worksheet, Hilbert was known for a thought experiment involving a hotel with a countable number of rooms (which showed up in Futurama). Assume Hilbert owns a hotel with a countable number of rooms, all of which are occupied, and answer the following:

1. How many rooms are in the hotel?
2. If one person shows up at Hilbert's door asking for a room, how could Hilbert accommodate him without throwing out any of his current guests?
3. All even numbered rooms need to be closed for maintenance. How can Hilbert avoid turning away any of his guests?
4. Say a set of people arrive at Hilbert's hotel (which is again full) and each would like their own room. Each person is uniquely identified by a rational number. Can Hilbert accommodate the new set of people without turning any of his current guests away?

Consider checking your answers with Examples 2 and 4 in §2.5 of your text.

5.