EXAM 2 TMath 308 Autumn 2014

NAME:

True/False: If the statement is *always* true, give a brief explanation of why it is. If the statement is false, give a counterexample.

1. [4] Given any matrices A, B, then $A + B$ is well defined.

2. [4] Given any matrices A, B, then $A * B = B * A$.

3. [4] The main diagonal of a skew-symmetric matrix must consist of entirely of zeros.

4. [4] Given any matrices A and B such that $A * B$ equals the zero matrix, then either A or B must be the zero matrix.

5. [4] Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}$. The vector $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ is in null(A).

6. [4] The columns in a 3×3 matrix A, form a basis for col(A).

Free Response: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

7. Let a, b, and c be real, nonzero numbers and let A, B, and C be matrices defined by:

$$A = \begin{bmatrix} a & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 1 \\ b & 0 \\ 3 & 0 \end{bmatrix}, \text{ and } C = \begin{bmatrix} c & 0 \\ 2 & 1 \end{bmatrix}.$$

Compute the following if possible.

(a) [2] $A^{\mathrm{T}} + B$

(b) [4] CA

(c) [3] $C^3 B$

(d) [3] C^{-1}

8. [4] Let $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Find the conditions on a, b,c and d such that M commutes multiplicatively with the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

9. [4] Consider the set $S = \{[x, y, z] \in \mathbb{R}^3 | z = 2x \text{ and } y = 0\}$.
Prove S is a subspace of $\mathbb{R}^3$ or give a counterexample to show that it does not.

10. Provide a lecture on induction. That is:

    (a) [3] explain the logic used in an induction proof,

    (b) [1] identify the essential elements in an induction proof, and

    (c) [2] provide an example induction proof with the above essential elements.