EXAM 1 # TMath 308 Fall 2014

NAME:

True/False: If the statement is *always* true, give a *brief* explanation of why it is (not a "formal" proof!). If the statement is false, give a counterexample.

1. [4] If $\overrightarrow{u}$, $\overrightarrow{v}$, and $\overrightarrow{w}$ are in $\mathbb{R}^n$, where $n \geq 2$, then $\overrightarrow{u} \cdot \overrightarrow{v} + \overrightarrow{w}$.

2. [4] The plane defined by $x + 2y + 3z = 1$ is parallel to the line defined by $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} t + \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ where $t \in \mathbb{R}$.

3. [4] If $\overrightarrow{u}$, $\overrightarrow{v}$, and $\overrightarrow{w}$ are in $\mathbb{R}^n$, where $n \geq 2$, then $\overrightarrow{u} \cdot (\overrightarrow{v} + \overrightarrow{w}) = \overrightarrow{u} \cdot \overrightarrow{v} + \overrightarrow{u} \cdot \overrightarrow{w}$.

4. [4] If a system of linear equations has more variables than equations, then the system has infinitely many solutions.

5. [4] The vectors $\overrightarrow{u}$, $\overrightarrow{v}$, and $\overrightarrow{w}$ are in $\text{span}(\overrightarrow{u}, \overrightarrow{v}, \overrightarrow{u} + \overrightarrow{w})$.

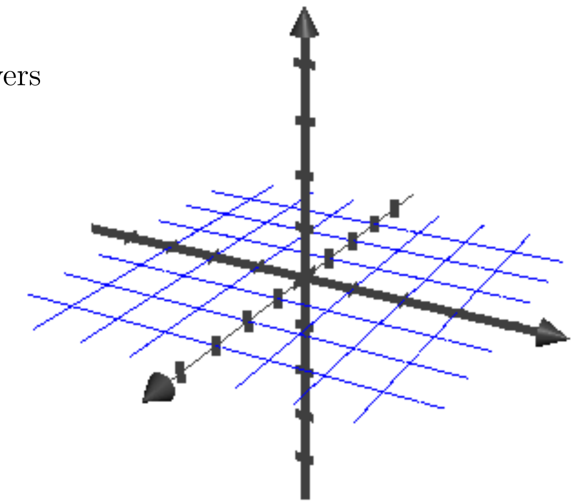
6. [4] The vector $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

Free Response: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

7. (a) [1] Identify the point $P = (-2, 4, 1)$ on the axis provided.

   (b) [1] Identify the vector $\overrightarrow{v} = \langle -2, 4, 1 \rangle$.

   (c) [2] Describe the difference between your answers for part (a) and (b).



   (d) [2] Compute $\overrightarrow{v} + 3\,\overrightarrow{i}$.

   (e) [2] Write down the equation of a line parallel to $\overrightarrow{v}$ that passes through P.

8. [2] Create a system of linear equations that has only one solution.

9. [6] Let a and b be nonzero real numbers and consider the augmented matrix: $\begin{bmatrix} 3 & 6 & 0 & -3 & : 18 \\ 0 & 0 & b & -6 & : 3 \\ a & 0 & -b & 0 & : 6a \end{bmatrix}$

   Use Gaussian Elimination, or Gauss-Jordan Elimination, or any series of elementary row operations that make sense to solve the system.

10. [2] Let a and b be nonzero real numbers and solve the following system of linear equations:

$$3x + 6y - 3w = 18$$

$$bz - 6w = 3$$

$$ax - bz = 6$$

11. [3] Identify a topic that did not appear on this exam and then construct a question about/for this topic. (Yes, I'd like you to help me write your final exam!)

12. [5] *Prove* $\text{proj}_{\overrightarrow{u}}(\text{proj}_{\overrightarrow{u}}(\overrightarrow{v})) = \text{proj}_{\overrightarrow{u}}(\overrightarrow{v})$ where $\overrightarrow{u}$ and $\overrightarrow{v}$ are in $\mathbb{R}^n$.