FINAL

TQS 308

Fall 2010

True/False: If the statement is *always* true, give a brief explanation of why it is. If the statement is false, give a counterexample.

1. [4] Let $\overrightarrow{u}$ and $\overrightarrow{v}$ be vectors in $\mathbb{R}^n$, $\overrightarrow{u} + \overrightarrow{v} = \overrightarrow{v} + \overrightarrow{u}$.

2. [4] Let l be the line that passes through the point $(1, -1, 1)$ and has a direction vector $\overrightarrow{d} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$. The line l is parallel to the plane defined by $2x + 3y - z = 1$.

3. [4] Any line in $\mathbb{R}^2$ is a subspace.

4. [4] A set of vectors that are linearly independent in $\mathbb{R}^3$ also form a basis for $\mathbb{R}^3$.

5. [4] For any matrices A and B, $\det(A + B) = \det(A) + \det(B)$.

6. [4] All linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$ have a nonzero eigenvector.

Free Responce: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.
(If you use a calculator, be sure to tell me.)

7. [3] Find all solutions to the following system of linear equations: $\left\{ \begin{array}{r} x_1 - 3x_2 - 2x_3 = 0 \\ -x_1 + 2x_2 + x_3 = 0 \\ 2x_1 + 4x_2 + 6x_3 = 0 \end{array} \right\}$

8. [2] Let $M = \begin{bmatrix} 1 & -3 & -2 \\ -1 & 2 & 1 \\ 2 & 4 & 6 \end{bmatrix}$. Find a basis for Null$(M)$.
Hint: consider using your work from the previous question.

9. [2] Let $A = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $B = \begin{bmatrix} -3 \\ 2 \\ 4 \end{bmatrix}$, and $C = \begin{bmatrix} -2 \\ 1 \\ 6 \end{bmatrix}$. Determine if A, B, and C span $\mathbb{R}^3$.
Justify your result. *Hint: consider using your work from the previous questions.*

10. Let $N = \begin{bmatrix} 0 & 0 & c \\ 0 & b & -c \\ a & a & 0 \end{bmatrix}$, $P = \begin{bmatrix} 1 & 0 & a \\ 0 & f & a \end{bmatrix}$, and $Q = \begin{bmatrix} 1 & 0 \\ 0 & f \\ f & 0 \end{bmatrix}$, where a, b, c, and f are nonzero real numbers. Find the following if possible:

(a) [1] $P + Q^{\mathrm{T}}$

(b) [1] NP

(a) [2] PQ

(b) [3] N^{-1}

11. [4] Let $\overrightarrow{v}$ be a vector in $\mathbb{R}^n$, prove $||\overrightarrow{v}|| = 0$ if and only if $\overrightarrow{v} = \overrightarrow{0}$.

12. Let k be a nonzero real number. Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T\overrightarrow{v} = A\overrightarrow{v}$, where $A = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$.

(a) [3] Describe geometrically the effect of the matrix transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$.

(b) [1] What is the characteristic equation for A?

(c) [2] Either use the above work or geometry to find the eigenvalues and an associated eigenvector for A.

(d) [2] Find the matrix that would record the following series of linear transformations on $\mathbb{R}^2$ with matrix multiplication:

i. apply the linear transformation T

ii. reflect over the y-axis.