FINAL

TQS 308

Fall 2009

True/False: If the statement is *always* true, provide a proof of why it is. If the statement is false, give a counterexample.

1. [4] The angle between $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} \sqrt{3}-1 \\ \sqrt{3}+1 \end{bmatrix}$ is $60°$.

2. [4] For every vector $\overrightarrow{x}$ in $\mathbb{R}^n$ and scalar c, $||c\overrightarrow{x}|| = c||\overrightarrow{x}||$.

3. [4] If $A = \begin{bmatrix} 2 & -1 & 0 & 3 \\ 4 & -2 & 1 & 3 \end{bmatrix}$, then $\begin{bmatrix} 1 & 2 & 0 & 0 \end{bmatrix}^\top$ is in Null(A).

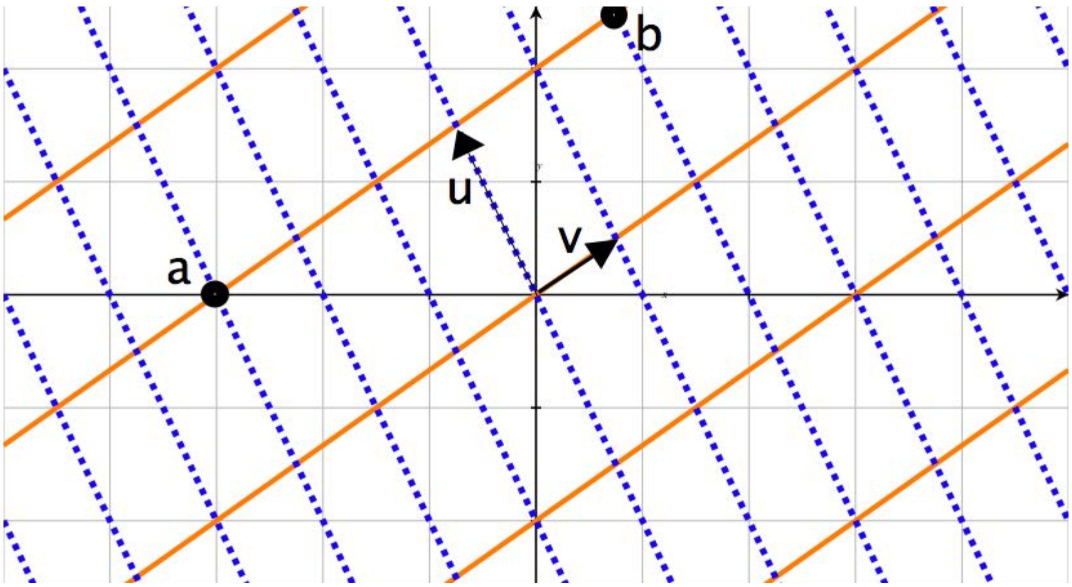
4. [4] If $V = \text{Span}(\overrightarrow{v_1}, \overrightarrow{v_2}, ...\overrightarrow{v_n})$, then $\{\overrightarrow{v_1}, \overrightarrow{v_2}, ...\overrightarrow{v_n}\}$ forms a basis of V.

5. [4] The vector $\overrightarrow{x} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ is an eigenvector of $A = \begin{bmatrix} 7 & 1 & -2 \\ -3 & 3 & 6 \\ 2 & 2 & 2 \end{bmatrix}$ with a corresponding eigenvalue of 6.

6. [4] If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ then $A^n = \begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix}$ for all $n \geq 1$.

Free Response: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

1. Let $\vec{v}$ and $\vec{u}$ be vectors from $\mathbb{R}^2$ depicted below.



   (a) [2] Label the point in $\mathbb{R}^2$ that corresponds to $\vec{u} - 2\vec{v} = \vec{z}$.

   (b) [2] Let $U = \text{Span}(\vec{u})$. Identify U on $\mathbb{R}^2$ above.

   (c) [3] Write the vector form of the equation of the line that passes through a and b. Do not approximate $\vec{u}$ or $\vec{v}$ but use them if needed in the expression.

   (d) [2] Notice that $\vec{u}$ and $\vec{v}$ span $\mathbb{R}^2$. Write $\vec{a}$ as a linear combination of $\vec{u}$ and $\vec{v}$.

   (e) [4] Is U a subspace? Justify your conclusion.

2. Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be a function that projects all the points in $\mathbb{R}^2$ onto the line $t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, where $t \in \mathbb{R}$.

   (a) [4] Write down a matrix that acts on the point (x,y) in the same way that F does.

   (b) [3] Is F invertible? If so, find F^{-1}. If not, explain why.

3. Let $A = \begin{bmatrix} -1 & 0 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

(a) [4] Find the characteristic polynomial of A.

(b) [7] Find all the eigenvalues and a *basis* for their corresponding eigenspaces for the matrix A.