EXAM 2 | TMath 308 | Fall 2013

NAME:

True/False: If the statement is *always* true, give a brief explanation of why it is. If the statement is false, give a counterexample.

1. [80] The vector $\overrightarrow{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is in the span $\left(\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right)$.

2. [80] Given matrices A, B, and C and that $A * B = A * C$, then $B = C$.

3. [80] If A and B are both 2×2 matrices, then $(A + B)^{-1} = A^{-1} + B^{-1}$.

4. [80] The set $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ is linearly independent.

5. [80] If A and B are symmetric $n \times n$ matrices, then AB is symmetric.

6. [80] The set $\{(x, y, z) \in \mathbb{R}^3 | x - y + z = 1\}$ is a subspace.

Free Responce: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

7. Let A, B, and C be defined by:

$$A = \begin{bmatrix} 3 & 0 & 0 \\ -1 & 5 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 4 & 1 \\ 0 & 0 \\ 3 & 0 \end{bmatrix}, \text{ and } C = \begin{bmatrix} -2 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Compute the following if possible.

(a) [60] $A + B^{\mathrm{T}}$

(b) [40] CA

(c) [60] a basis for $\operatorname{col}(A)$

(d) [80] C^{-1}

8. [60] Given that A, B, and X are all matrices of appropriate sizes, solve for the matrix X. Assume all matrices are invertible and simplify as much as possible.

$$ABXA^{-1}B^{-1} = I + A$$

9. [80] Prove that if AB and BA are both defined then AB and BA are both square matrices.

10. Let W be the set of all vectors $\overrightarrow{u} \in \mathbb{R}^2$ that exhibit the property:

$$||\overrightarrow{u} + \begin{bmatrix} -1 \\ 1 \end{bmatrix}|| = ||\overrightarrow{u}|| + ||\begin{bmatrix} -1 \\ 1 \end{bmatrix}||.$$

In set builder notation:

$$W = \left\{\overrightarrow{u} \in \mathbb{R}^2; ||\overrightarrow{u} + \begin{bmatrix} -1 \\ 1 \end{bmatrix}|| = ||\overrightarrow{u}|| + ||\begin{bmatrix} -1 \\ 1 \end{bmatrix}||\right\}.$$

(a) [60] Describe geometrically what vectors the set W contains. (A picture is worth a thousand words and no proof is needed.)

(b) [80] Either prove W is a subspace or determine which subspace property is not satisfied by W.