EXAM 1 # TMath 308 Fall 2013

NAME:

True/False: If the statement is *always* true, give a *brief* explanation of why it is (not a formal proof!). If the statement is false, give a counterexample.

1. [4] Let $A = (3, 4, 5)$, $B = (2, 4, 2)$, $C = (-1, 0, -3)$ be points in $\mathbb{R}^3$ and let O denote the origin. Then the vector $\overrightarrow{AB}$ is "the same" (i.e. equivalent to the) vector as $\overrightarrow{CO}$.

2. [4] For every vector $\overrightarrow{u}$ and $\overrightarrow{v}$ in $\mathbb{R}^n$ and scalar $c \in \mathbb{R}$

$$c(\overrightarrow{u} + \overrightarrow{v}) = c\overrightarrow{u} + c\overrightarrow{v}.$$

3. [4] The vectors $||\overrightarrow{u} + \overrightarrow{v}|| = ||\overrightarrow{u}|| + ||\overrightarrow{v}||$ if and only if $\overrightarrow{u} = \overrightarrow{0}$.

4. [4] The following is the normal form of a line in $\mathbb{R}^3$, where $\begin{bmatrix}1\\1\\1\end{bmatrix}$ is perpendicular to the line.

$$\begin{bmatrix}1\\1\\1\end{bmatrix} \cdot \left(\begin{bmatrix}x\\y\\z\end{bmatrix} - \begin{bmatrix}3\\6\\9\end{bmatrix} \right) = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

5. [4] The vector $\begin{bmatrix}5\\1\end{bmatrix}$ is a linear combination of $\begin{bmatrix}1\\-1\end{bmatrix}$ and $\begin{bmatrix}1\\1\end{bmatrix}$

6. [4] Every homogenous system of linear equations has at least one solution.

Free Response: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

7. [7] The following problem is from the chapter on rectangular arrays in the Chiu Chang Suan Shu (Nine Chapters on the Mathematical Art) text written before 100CE. Solve it and be sure to show your work so I can follow your steps!

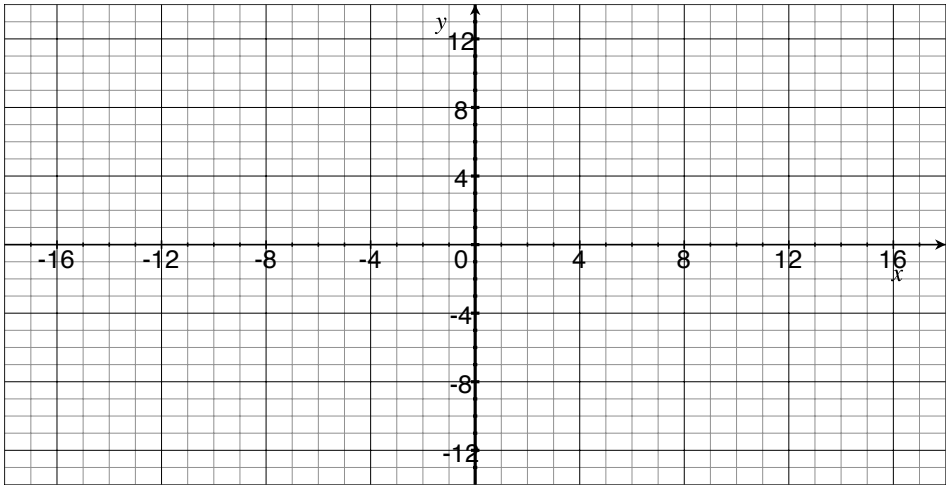
   > The total yield of 3 sheaves of superior grain, 2 sheaves of medium grain, and 1 sheaf of inferior grain is 39 *duo* of rice. The total yield of 2 sheaves of superior grain, 3 sheaves of medium gran, and 1 sheaf of inferior grain is 34 *duo*. The total yield of 1 sheaf of superior grain, 2 sheaves of medium grain, and 3 sheaves of inferior grain is 26 *duo*. What is the yield of one sheaf of each grade of grain?

8. [3] A lawn mower has a mass of 30kg. It is being pushed with a force of 100N. If the handle of the lawn mower makes an angle of 40° with the ground, what is the horizontal component of the force that is causing the mower to move forward?

9. Let the following vectors be in $\mathbb{R}^2$.

$$\overrightarrow{u} = [-2, 5] \qquad \overrightarrow{v} = [2, 2] \qquad \overrightarrow{w} = [0, 2]$$

(a) [3] Draw the vector $\overrightarrow{u} + 2\overrightarrow{v} - \overrightarrow{w}$ on the axes below.



(b) [3] Find the angle between $\overrightarrow{u}$ and $\overrightarrow{v}$. Specify if you are using degrees or radians.

10. [5] Let $\mathcal{P}_1$ and $\mathcal{P}_2$ be the planes defined by $3x - 2y + z = -1$ and $\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 5$ respectively. Write out the steps (done by hand) that you would take to find the line of intersection of the two planes. Note, you do *not* need to follow the steps yourself! You just need to write them out so an intern (who knows how to find the row echelon and reduced row echelon form) could follow the steps.

11. [5] *Prove* $\text{proj}_{\overrightarrow{u}}\left(\overrightarrow{v} - \text{proj}_{\overrightarrow{u}}(\overrightarrow{v})\right) = \overrightarrow{0}$ where $\overrightarrow{u}$ and $\overrightarrow{v}$ are in $\mathbb{R}^n$.