Key /62

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1) False (+2) let $A=\begin{bmatrix}0&0\\0&0\end{bmatrix}$, $B=\begin{bmatrix}1&2\\3&4\end{bmatrix}$, and $C=\begin{bmatrix}5&6\\7&8\end{bmatrix}$

looking for ex (+1) got it (+1)

Note $AB=AC$ but $B\neq C$.

[A more interesting ex ie when A isn't 0 is:

$A=\begin{bmatrix}-2&-2\\-2&-1\end{bmatrix}$ $B=\begin{bmatrix}1&1\\2&2\end{bmatrix}$ and $C=\begin{bmatrix}3&3\\6&6\end{bmatrix}$]

2) False (+2) if $CA=B$ then $CAA^{-1}=BA^{-1}$

$\Rightarrow CI=BA^{-1} \Rightarrow C=BA^{-1}$

matrix mult is not commutative so BA^{-1} may not equal A^{-1}

started (+1) got a reason (+1)

3) True (+2) $(A-B)(A+B)=A(A+B)-B(A+B)$ by dist prop

$=A^2+AB-BA-B^2$ by dist prop

(+1) started w/ assump (+1) got it

$=A^2+(AB-BA)-B^2$

$=A^2+0-B^2$ by assumption

$=A^2-B^2$.

4) False (+2) $2\begin{bmatrix}-2\\1\\2\end{bmatrix}+1\begin{bmatrix}4\\-1\\-1\end{bmatrix}=\vec{0}$ so the set is not lin. indep

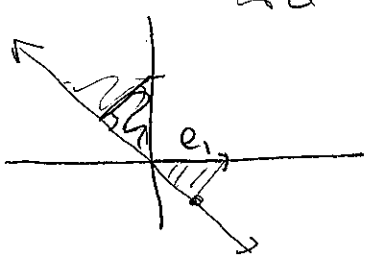
started (+1) reason (+1)

5) False (+2) the rank thm is $rk(A)+null(A)=n$.

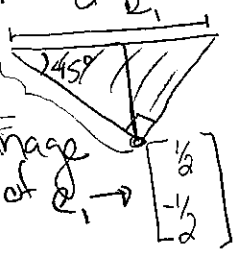
ex $\begin{bmatrix}1&0&0\\0&1&0\end{bmatrix}$ has rk 2 and nullity 1

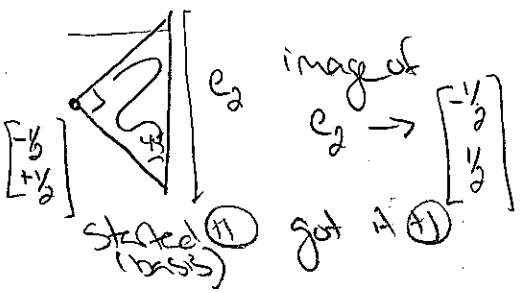
[basis is $\left\{\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$]

6) True (+2)



and $2+1*2$





started (+1) got it (+1)

a) $-B = \begin{bmatrix} -3 & 0 & -1 \\ 0 & -2 & 0 \end{bmatrix}$

scale (+1) got it (+1)

b) BA 2x3 times 2x2

not possible (+1)
started (+1)

c) $CB^T = \begin{bmatrix} -2 & 3 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{3\times3} \begin{bmatrix} 3 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}^{3\times2}$ (+1)

$= \begin{bmatrix} -6 & 6 \\ 6 & 2 \\ 1 & 0 \end{bmatrix}$ (+1)

d) $\det(A) = 8 - 8 = 0$

$\Rightarrow A^{-1}$ does not exist.

started (+1) got it (+1)

e) basis for row A.

row reductions (+1)

$\begin{bmatrix} 8 & 2 \\ 4 & 1 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \frac{1}{4} \\ 0 & 0 \end{bmatrix}$

basis: $\{ [4 \ 1] \}$ (+1) no more (+1)

f) basis for null A.

let $x_2 = s$ then $x_1 = -\frac{1}{4}s$ param (+1)

Sol set to homogeneous system (+1)

$\begin{bmatrix} -\frac{1}{4}s \\ s \end{bmatrix} = s \begin{bmatrix} -\frac{1}{4} \\ 1 \end{bmatrix}$

basis $= \left\{ \begin{bmatrix} -\frac{1}{4} \\ 1 \end{bmatrix} \right\}$ (+1)

g) $\left[\begin{array}{ccc|ccc} -2 & 3 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2+R_1} \left[\begin{array}{ccc|ccc} -2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$ alg (+1)

(+1)

$\rightarrow \left[\begin{array}{ccc|ccc} 1 & -\frac{3}{2} & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{8} & \frac{3}{8} & 0 \\ 0 & 1 & 0 & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

$\frac{3}{2} \cdot \frac{1}{4} = \frac{3}{8}$

$-\frac{4}{8}$

OK or know right etc (+1)

h) work is above

form (+1)

OK/right etc (+1)

$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} -2 & 3 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

stated (+1)

2) Claim (+1) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for $n \geq 1$.

form (+1)

BC: $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ ✓ (+1)

induction: Assume $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ (+1)

we want to show $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{k+1} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}$

note $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^k$ by def

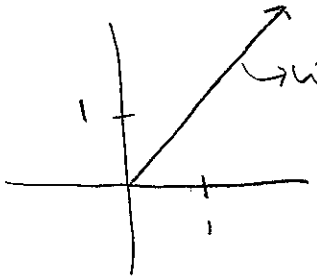
$= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ by assump (+1) use assump

logic (+1)

$= \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}$ by matrix mult

3) a) W is the set of all vectors in standard position on the positive side of the line $y=x$. (+1) (+1)

started (+1)



b) W is not a subspace, it is not closed under scalar mult. (+1)

Note $\begin{bmatrix}1\\1\end{bmatrix} \in W$ (+1) since $\left\|\begin{bmatrix}1\\1\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}\right\| = \left\|\begin{bmatrix}2\\2\end{bmatrix}\right\| = \sqrt{4+4}$

comp (+1)

$= \sqrt{8} = 2\sqrt{2} = \sqrt{2}+\sqrt{2}$

$= \left\|\begin{bmatrix}1\\1\end{bmatrix}\right\| + \left\|\begin{bmatrix}1\\1\end{bmatrix}\right\|$

but $-1\cdot\begin{bmatrix}1\\1\end{bmatrix} \notin W$ (+1) since $\left\|-1\begin{bmatrix}1\\1\end{bmatrix}+\begin{bmatrix}1\\1\end{bmatrix}\right\| = \left\|\begin{bmatrix}0\\0\end{bmatrix}\right\| = 0$

$\neq 2\sqrt{2} = \sqrt{2}+\sqrt{2}$

$= \left\|-1\begin{bmatrix}1\\1\end{bmatrix}\right\| + \left\|\begin{bmatrix}1\\1\end{bmatrix}\right\|$

(+1) cond of subspace

(+1) rb ~~tring~~ elements

(+1) assume element is $\in W$ & show closure (+1)

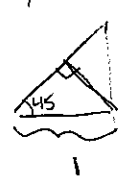
show close

c) $\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x\\-y\end{bmatrix}$, (+1)

The transformation flips all the points in $\mathbb{R}^2$ over the x-axis. (+1)

EC) d) the proj. onto the line $y=x$ would work

so $\begin{bmatrix}1/2 & 1/2\\1/2 & 1/2\end{bmatrix}$



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