MIDTERM 2 # TQS 308 Fall 2009

True/False: If the statement is *always* true, give a brief explanation of why it is. If the statement is false, give a counterexample.

1. [4] If A, B, and C are 2×2 non-zero matrices and $AB = AC$, then $B = C$.
2. [4] If A, B, and C are invertible matrices such that $CA = B$, then $C = A^{-1}B$.
3. [4] If A and B are square matrices so that $AB = BA$, then
$$(A - B)(A + B) = A^2 - B^2.$$
4. [4] The set $\left\{ \begin{bmatrix} -2 \\ \frac{1}{2} \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^2$.
5. [4] Given an $m \times n$ matrix A, $\text{rk}(A) + \text{null}(A) = m$.
6. [4] The transformation T from $\mathbb{R}^2$, to $\mathbb{R}^2$ that projects $\mathbb{R}^2$ onto the line $y = -x$ is given by the rule:
$$\overrightarrow{x} \to \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \overrightarrow{x}.$$

Free Response: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

1. Let
$$A = \begin{bmatrix} 8 & 2 \\ 4 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} -2 & 3 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$
Find the following if possible.
(a) [2] $-B$
(b) [2] BA
(c) [2] CB^{T}
(d) [2] A^{-1}
(e) [3] a basis for row(A)
(f) [3] a basis for null(A)
(g) [3] C^{-1}
(h) [3] the LU factorization of C.
2. [7] Use induction to prove a general formula for A^n $(n \geq 1)$ if $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

3. Let W be the set of all vectors $\overrightarrow{u} \in \mathbb{R}^2$ that exhibit the property:

$$||\overrightarrow{u} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}|| = ||\overrightarrow{u}|| + ||\begin{bmatrix} 1 \\ 1 \end{bmatrix}||.$$

In set builder notation:

$$W = \left\{ \overrightarrow{u} \in \mathbb{R}^2; ||\overrightarrow{u} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}|| = ||\overrightarrow{u}|| + ||\begin{bmatrix} 1 \\ 1 \end{bmatrix}|| \right\}.$$

(a) [3] Describe geometrically what vectors the set W contains. (A picture is worth a thousand words and no proof is needed.)

(b) [4] Either prove W is a subspace of find and exhibit which subspace property is not satisfied by W.

(c) [2] Let S be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ that sends $\overrightarrow{x}$ to $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \overrightarrow{x}$. Describe in words what S does to W.

(d) [Extra Credit 2] Find a matrix that acts nontrivially on $\mathbb{R}^2$ (some points in $\mathbb{R}^2$ are not sent back to themselves) but returns W to itself.