MIDTERM 1 **TQS 308** Fall 2009

NAME:

True/False: If the statement is *always* true, give a brief explanation of why it is. If the statement is false, give a counterexample.

1. [4] For every vector $\overrightarrow{u}$ and $\overrightarrow{v}$ in $\mathbb{R}^n$

$$\overrightarrow{u} + \overrightarrow{v} = \overrightarrow{v} + \overrightarrow{u}.$$

2. [4] For every vector $\overrightarrow{u}$, $\overrightarrow{v}$, and $\overrightarrow{w}$ if $\overrightarrow{u} \cdot \overrightarrow{v} = \overrightarrow{u} \cdot \overrightarrow{w}$, then $\overrightarrow{v} = \overrightarrow{w}$.

3. [4] For every vector $\overrightarrow{u}$, $\overrightarrow{v}$, and $\overrightarrow{w}$

$$(\overrightarrow{u} \cdot \overrightarrow{v}) \cdot \overrightarrow{w} = \overrightarrow{u} \cdot (\overrightarrow{v} \cdot \overrightarrow{w}).$$

4. [4] For every vector $\overrightarrow{v}$ in $\mathbb{R}^n$ and scalar c, $||c\overrightarrow{v}|| = c||\overrightarrow{v}||$.

5. [4] $2 * 3 + 1 = 1 \mod 5$.

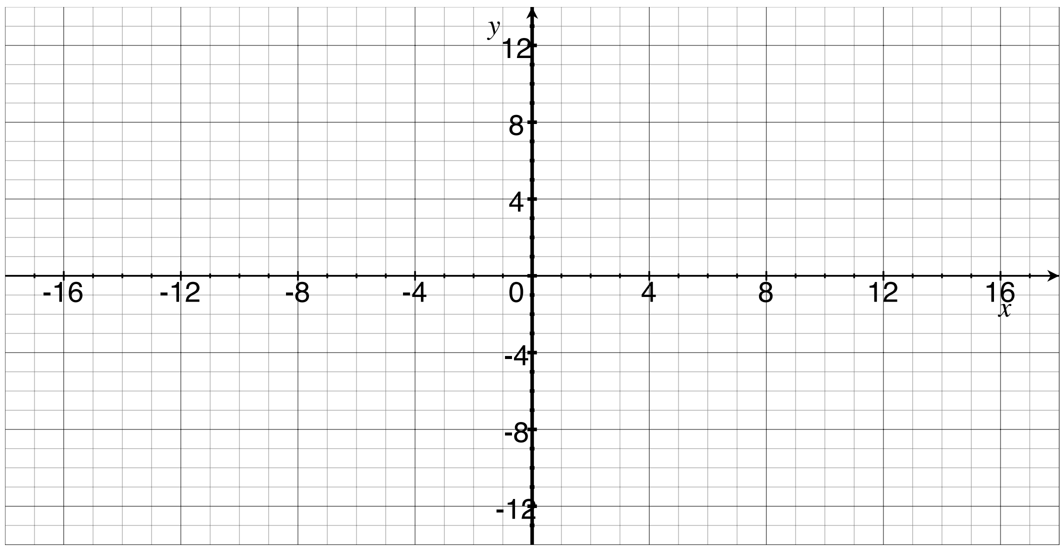
6. [4] Every system of linear equations has a solution.

Free Responce: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

7. Let the following vectors be in $\mathbb{R}^2$.

$$\overrightarrow{u} = [0,7] \qquad \overrightarrow{v} = [4,-4] \qquad \overrightarrow{w} = [8,-1]$$

(a) [3] Draw $\overrightarrow{u} - \overrightarrow{v}$ on the axes below.



(b) [3] Find the tail if the vector $\overrightarrow{w}$ (in standard position) is translated so it's head is at the point $(-4,-8)$

(c) [6] Find the exact angle between $\overrightarrow{u}$ and $\overrightarrow{v}$.

(d) [5] Is $\overrightarrow{u}$ a linear combination of $\overrightarrow{v}$ and $\overrightarrow{w}$, if so write $\overrightarrow{u}$ as a linear combination of $\overrightarrow{v}$ and $\overrightarrow{w}$.

(e) [1] Verify your answer to (d) graphically.

8. Let the following vectors be in $\mathbb{R}^3$.

$$\overrightarrow{u} = [4, 3, -1] \qquad\qquad \overrightarrow{v} = [-1, 1, 0]$$

(a) [2] Write the vector form for a line that is parallel to $\overrightarrow{u}$ and passes through $(1, 2, 3)$.

(b) [5] Let $\mathcal{P}$ be the plane defined by $2x - 2y + 2z = 5$. Determine if $\mathcal{P}$ is parallel, perpendicular, or neither to $\overrightarrow{u}$.

(c) [5] Let $\mathcal{L}$ be the line that passes through the point $A = (3, 1, 1)$ and has a direction vector $\overrightarrow{v}$. Find the distance from the point $B = (1, 0, 2)$ to the line $\mathcal{L}$.

9. [8] Use Gaussian or Gauss-Jordan elimination to find solution(s) if they exist, to the following system of linear equations: $\left\{\begin{array}{c} x_1 - 3x_2 - 2x_3 = 0 \\ -x_1 + 2x_2 + x_3 = 0 \\ 2x_1 + 4x_2 + 6x_3 = 0 \end{array}\right\}$

10. [3] Find the reduced row echelon (or row echelon) form for A where $A = \begin{bmatrix} 1 & -3 & -2 & 0 \\ -1 & 2 & 1 & 0 \\ 2 & 4 & 6 & 0 \end{bmatrix}$

(a) [2] What is the rank of A?

11. [8] Find all vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ in $\mathbb{R}^3$ that exhibit the property:

$$||\overrightarrow{u} + \overrightarrow{v}|| = ||\overrightarrow{u}|| + ||\overrightarrow{v}||.$$

Be sure to show (prove) you've found *all* the vectors that work.