EXAM 2

TQS 308

Fall 2010

NAME:

True/False: If the statement is *always* true, give a brief explanation of why it is. If the statement is false, give a counterexample.

1. [4] If A and B are both 2×2 matrices, then $AB = BA$

2. [4] A matrix of the form $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ where a, $b \in \mathbb{R}$ commutes with $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

3. [4] If A and B are both 2×2 matrices, then $(A + B)^{-1} = A^{-1} + B^{-1}$.

4. [4] A basis for the subspace: $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + r \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} | r, t \in \mathbb{R} \right\}$ in $\mathbb{R}^3$ is: $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} \right\}$

5. [4] A set of vectors that are linearly independent in $\mathbb{R}^3$ also form a basis for $\mathbb{R}^3$.

6. [4] The transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ that embeds $\mathbb{R}^2$ into the xy-plane is given by

$$\overrightarrow{x} \longrightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \overrightarrow{x}.$$

Free Responce: Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

7. Let A, B, and C be defined by:

$$A = \begin{bmatrix} -2 & 2 & 0 \\ 4 & -4 & 1 \end{bmatrix},\ B = \begin{bmatrix} -3 \\ 6 \\ 9 \end{bmatrix},\ \text{and } C = \begin{bmatrix} -2 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Compute the following if possible.

(a) [1] $2B$

(b) [2] A^{T}

(c) [2] a basis for col(A)

(d) [4] C^{-1}

8. Let A be a 2×3 matrix, B be a 3×1 matrix, and let C be a 3×3 matrix.

(a) [1] What are the dimensions of the matrix BC, if it exists?

(b) [1] What are the dimensions of the matrix CA^{T}, if it exists?

(c) [1] What are the dimensions of the matrix C^8B, if it exists?

9. Assume all matrices A_i given below are multiplicatively invertible and of such a size that the multiplication makes sense.

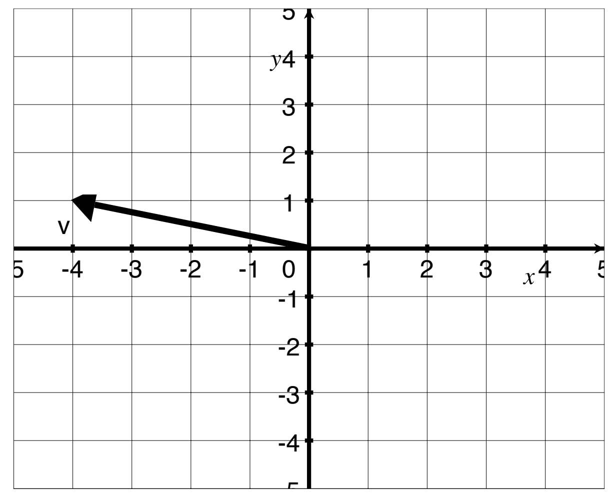
   (a) [2] What is the multiplicative inverse of A_2A_1?

   (b) [4] Use induction to prove that $A_n..A_2A_1$ is invertible for all n.

10. Consider the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T\overrightarrow{v} = A\overrightarrow{v}$, where

$$A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}.$$

(a) [1] The vector shown on the right is $\overrightarrow{v}$. Draw the image of $\overrightarrow{v}$ under the transformation T.



(b) [3] Find a basis for $null(A)$.

(c) [2] Identify $null(A)$ on the axes provided above.

(d) [2] Find any $\overrightarrow{x}$ that are *invariant* under the action of T. That is, find all $\overrightarrow{x}$ such that $T\overrightarrow{x} = \overrightarrow{x}$. Justify your answer either geometrically or algebraically.